The Incentive Compatibility Critique*

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Abstract

We present an IC (incentive compatibility) critique for the current mechanism design paradigm: in a classical mechanism design model, we prove that a mechanism designer facing a generic incomplete-information scenario can dispense with the incentive compatibility constraint without loss of generality. And hence, this "cast(s) doubt on the value of the current mechanism design paradigm as a model of institutional design" (McAfee and Reny, 1992, p. 400).

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1 Introduction

Starting with Akerlof (1970), Spence (1973), and Rothschild and Stiglitz (1976), a major theme in economic models has been *asymmetric information*. That is, every agent possesses private information, which is not accessible to other agents. In particular, an agent will disclose her private information only if it is beneficial for her to do so. This is the basic idea of the incentive compatibility condition (hereafter, IC) due to Hurwicz (1960, 1972).

IC is also the defining feature of the *mechanism design* literature, which focuses on how a social planner tries to design an institution to allocate limited resources. Without IC, the mechanism design problem becomes a trivial centralized allocation problem. It is trivial in the sense that any social choice function (hereafter, SCF) can be implemented.[1]

Given any SCF, there are usually two goals that a planner aims to achieve: implementation of the SCF and revenue maximization. Research to date suggests that in many economic situations, in order to respect IC, achieving even one of the two goals may be costly or impossible. For example, in an auction setting with independent types, Myerson (1981) show that privately-informed agents retain informational rent even under a revenue-maximizing mechanism, i.e., full surplus extraction cannot be achieved; in a bilateral trade setting with independent types, Myerson and Satterthwaite (1983) show that inefficiency is inevitable under budget-balanced transfers, i.e., the efficient SCF cannot be implemented in any mechanism that balances the budget.

In this paper, we focus on a classical mechanism design model which is adopted in many economic problems (e.g., auctions, bilateral trade, or public goods provision). We fix an SCF satisfying a mild condition called *the almost-continuous condition* (see Definition 1). The almost-continuous condition is satisfied by virtually all SCFs in the economic literature. We show that, surprisingly, for generic incomplete-information scenarios, we can implement the SCF and extract the full surplus, even if we respect IC. That is, generically, IC is dispensable in the sense that we are able to achieve the two goals with or without imposing IC.

More specifically, we consider a finite set of agents with quasilinear utilities. Each

[1] See Propositions 1 and 2 for formal arguments.

agent holds private information modeled by (common) priors on the universal type space (see Mertens and Zamir (1985) and Heifetz and Neeman (2006)). The universal type space is a (Harsanyi) type space which embeds all possible type spaces. Let $\mathcal{F}$ denote the set of all possible priors for the agents. In particular, let $\mathcal{F}^{IR,SE}$ denote the set of all possible priors on which we can implement the SCF while satisfying the individual rationality condition (hereafter, IR) and the full surplus extraction condition (hereafter, SE), i.e., we ignore IC (and other conditions such as budget balance) for $\mathcal{F}^{IR,SE}$. It is straightforward to see that

$$\mathcal{F} = \mathcal{F}^{IR,SE}$$

That is, the SCF can always be implemented without sacrificing informational rent if IC is ignored. Let $\mathcal{F}^{IC,IR,SE}$ denote the set of all possible priors, on which we can implement the SCF with IC, IR, and SE satisfied. Then, our main result is that $\mathcal{F}^{IC,IR,SE}$ is a generic subset of $\mathcal{F}^{IR,SE}$. That is, IC has bites only in a "negligible" set of priors, i.e., generically, we can dispense with IC without loss of generality. This is the IC critique.

The IC critique is closely related to the FSE (full surplus extraction) critique discussed in Crémer and McLean (1988), McAfee and Reny (1992), Heifetz and Neeman (2006) and Chen and Xiong (2013a). To draw the connection, consider a private-value setup and the *efficient* SCF. Let $\mathcal{F}^{IC,IR}$ denote the set of all possible priors on which we can implement the efficient SCF with IC and IR satisfied. By invoking the VCG mechanism, we see that

$$\mathcal{F} = \mathcal{F}^{IC,IR}.$$

Let $\mathcal{F}^{IC,IR,SE}$ denote the set of all possible priors on which we can implement the efficient SCF with IC, IR, and SE satisfied. The FSE critique says that $\mathcal{F}^{IC,IR,SE}$ is a generic subset of $\mathcal{F}^{IC,IR}$, i.e., generically, full surplus extraction can be achieved.

Despite the apparent difference between the IC critique and the FSE critique, they share a similar basic intuition: generically, the agents' private information is correlated. As shown in Crémer and McLean (1988), a mechanism designer can always utilize the correlation to design payment schemes to satisfy IC, IR and SE. We now illustrate the two critiques in an auction environment.

Example 1 (The FSE Critique) *An auction will be held to sell an object, and we wish to achieve* *the efficient outcome. Most likely, the agents' private values are at least slightly correlated, because they read the same newspapers; they received similar education; they have similar cultural backgrounds, etc., all of which may influence their valuation of the object for sale. No matter how slightly their values are correlated, theoretical results (e.g., Crémer and McLean (1988) and Chen and Xiong (2013a)) show that we can utilize the correlation to couple the Second-Price auction with a carefully designed payment scheme to achieve both the efficient outcome and full surplus extraction. This contradicts what we usually observe in reality: no practical mechanism (that we know of) makes use of such a correlation to fully extract informational rent. This is the FSE critique.*

Example 2 (The IC Critique) *An auction will be held to sell an object, and we wish to achieve an arbitrary outcome. Most likely, the agents' private values are at least slightly correlated for the reasons given above. No matter how slightly their values are correlated, theoretical results show that we can utilize the correlation to design a payment scheme to achieve both the desirable outcome (whether efficient or inefficient) and full surplus extraction. This contradicts what we usually observe in reality: no practical mechanism (that we know of) makes use of such a correlation to implement the outcome and/or fully extract informational rents. This is the IC critique.*

We draw two important distinctions between the FSE critique and the IC critique. First, the FSE critique focuses on the efficient SCF in a private-value setup, while the IC critique applies to any SCF, even those with interdependent values. The intuition behind the two critiques is the same, namely, that (value) correlation is generic. Nevertheless, correlation plays different roles in the two critiques. It is well known that in the private-value setup, the VCG mechanism implements the efficient SCF (with IC satisfied). Thus, for the efficient SCF, correlation is utilized *only* to achieve SE, and hence we get the FSE critique. However, for an arbitrary SCF, we use correlation to achieve IC besides SE, which leads to the IC critique.

Second, as we know of no mechanism design problem that does not impose IC, IC is more fundamental than any other property in mechanism design.[2] For instance, IR

[2] Given a social choice to implement, a mechanism design problem is defined by the properties that the targeted mechanism should satisfy, e.g., IC, IR, SE. We say that a property is more fundamental than another if and only if the former is required whenever the latter is required in a mechanism design problem.

and SE are not required in the balanced mechanism design literature that dates back to Arrow (1979) and d'Aspremont and Gérard-Varet (1979) (see also d'Aspremont, Crémer, and Gérard-Varet (2003) for a survey). In Section 3.2, we show that the IC critique also applies to these settings.

(McAfee and Reny, 1992, p. 400) argue that

> "The results (full rent extraction) cast doubt on the value of the current mechanism design paradigm as a model of institutional design" (McAfee and Reny, 1992, p. 400).

Since IC is a more fundamental requirement than SE, the IC critique points to a more fundamental problem of the current mechanism design paradigm.

The rest of the paper formalizes the IC critique. Section 2 defines the model; Section 3 presents the main result; Section 4 concludes. All proofs are relegated to the appendix.

2 The model

2.1 Environments

Let I denote a finite set of agents and assume that $|I| \geq 2$. Let V_i denote the nonempty compact metric space of payoff types of agent i. Let $V = \Pi_{i\in I} V_i$ denote the set of payoff-relevant states. At every state $v = (v_i)_{i\in I} \in V$, each agent i privately observes her payoff type $v_i \in V_i$. Let X denote the set of social outcomes and assume that X is a metric space with metric ρ. Every agent i is risk-neutral and endowed with a continuous Bernoulli utility function $u_i : V \times X \to \mathbb{R}$, i.e., given $v \in V$, agent i gets utility $u_i(v, x)$ if outcome x is chosen. Thus, we allow for interdependent and multidimensional values. The setup also allows for stochastic allocations. For example, in single-object auctions, we may set $X = \left\{(x_i)_{i\in I} \in [0,1]^I : \sum_{i\in I} x_i \leq 1\right\}$, where x_i denotes the probability of agent i getting the object. For any probability distribution μ, let suppμ denote the support of μ, i.e., suppμ is the smallest closed set that, according to μ, has probability 1.

2.2 Belief subspaces

Let Θ_i^* be the compact, metric, universal type space on V that is constructed in (Heifetz and Neeman, 2006, pp. 228-229). Each $\theta_i \in \Theta_i^*$ is called a type of agent i. With slight abuse of notations, let $v_i : \Theta_i^* \to V_i$ and $b_i : \Theta_i^* \to \Delta\left(\Theta_{-i}^*\right)$ be the continuous functions through which each type $\theta_i \in \Theta_i^*$ identifies a payoff type $v_i\left(\theta_i\right)$ and a belief $b_i\left(\theta_i\right)$ over the types of agent i's opponents, respectively. The mappings $v_i\left(\cdot\right)$ and $b_i\left(\cdot\right)$ are commonly known to the agents. Let $\Theta^* = \Pi_{i \in I}\Theta_i^*$ be the space of type profiles of all agents, and let $\Theta_{-i}^* = \Pi_{j \neq i}\Theta_j^*$ be the space of type profiles of agent i's opponents. Let d_i, d_{-i} and d denote the metrics on Θ_i^*, Θ_{-i}^* and Θ^*, respectively, such that $d_{-i}\left(\theta_{-i}, \theta_{-i}'\right) = \max_{j \neq i} d_j\left(\theta_j, \theta_j'\right)$ and $d\left(\theta, \theta'\right) = \max_{j \in I} d_j\left(\theta_j, \theta_j'\right)$. For each $\theta \in \Theta^*$, we denote by θ_i the type of agent i under θ, and we may simplify the notation by writing $v_i\left(\theta\right)$ and $b_i\left(\theta\right)$ instead of $v_i\left(\theta_i\right)$ and $b_i\left(\theta_i\right)$, respectively.

Definition 1 *A belief subspace $\Theta = \Theta_i \times \Theta_{-i}$ is a nonempty and compact subset of Θ^* such that for every $\left(i, \theta\right) \in I \times \Theta$, $\left\{\theta_i\right\} \times Suppb_i\left(\theta\right)$ is a subset of Θ.*

2.3 Mechanisms and implementation

Our goal is to use a mechanism to implement a given social choice function (SCF) $f : V \to X$. Throughout the paper, we fix an SCF f that is *almost-continuous* in the following sense.

Assumption 1 (Almost-Continuous SCF) *$\left\{v \in V : f \text{ is continuous at } v\right\}$ is a dense set in V.*

Put another way, f is almost-continuous in our sense if, whenever f is discontinuous at v, there is some v' arbitrarily close to v such that f is continuous at v'. Virtually all SCFs considered in economic models are almost-continuous. For example, in private-value, single-object auctions, the efficient SCF that always assigns the object to an agent with the highest value is almost-continuous.

By the revelation principle, we can restrict attention to direct mechanisms. A (direct) mechanism on a belief subspace Θ is a list of measurable functions $q : \Theta \to X$ and

$(m_i : \Theta \to \mathbb{R})_{i \in I}$, where, at each $\theta \in \Theta$, $q(\theta)$ specifies the outcome and $m_i(\theta)$ specifies how much agent i pays.

For any mechanism (q, m) defined on a belief subspace Θ, let $u_i\left(\theta_i', \theta_{-i} | \theta_i, q, m\right)$ denote the ex post payoff of type θ_i, when he reports θ_i' and the other agents truthfully report their types; more formally,

$$u_i\left(\theta_i', \theta_{-i} | \theta_i, q, m\right) \equiv u_i\left(v\left(\theta_i, \theta_{-i}\right), \, q\left(\theta_i', \theta_{-i}\right)\right) - m_i\left(\theta_i', \theta_{-i}\right). \tag{1}$$

Furthermore, let $U_i\left(\theta_i' | \theta_i, q, m\right)$ denote the interim expected payoff of type θ_i when he reports θ_i' and the other agents truthfully reveal their types; more formally,

$$U_i\left(\theta_i' | \theta_i, q, m\right) = \int_{\Theta_{-i}} u_i\left(\theta_i', \theta_{-i} | \theta_i, q, m\right) b_i\left(\theta_i\right)\left[d\theta_{-i}\right].$$

For simplicity, we write $U_i\left(\theta_i | q, m\right)$ for $U_i\left(\theta_i | \theta_i, q, m\right)$, which is the interim expected payoff of type θ_i when all agents truthfully reveal their types.

We now define the individual rationality condition (IR), the surplus extraction condition (SE), the (ex post) budget balance condition (BB), the incentive compatibility condition (IC), and the implementation condition (IM) conditions as follows:

Definition 2 *A mechanism (q, m) defined on a belief subspace Θ satisfies IR if*

$$U_i\left(\theta_i | q, m\right) \geq 0, \forall\left(i, \theta\right) \in I \times \Theta.$$

Definition 3 *For any $\varepsilon \geq 0$, a mechanism (q, m) defined on a belief subspace Θ satisfies ε-SE if*

$$U_i\left(\theta_i | q, m\right) \leq \varepsilon, \forall\left(i, \theta\right) \in I \times \Theta.$$

Definition 4 *A mechanism (q, m) defined on a belief subspace Θ satisfies BB if*

$$\sum_{i \in I} m_i\left(\theta\right) = 0, \forall \theta \in \Theta.$$

Definition 5 *For any $\varepsilon \geq 0$, a mechanism (q, m) defined on a belief subspace Θ satisfies ε-IC if*

$$U_i\left(\theta_i | q, m\right) \geq U_i\left(\theta_i' | \theta_i, q, m\right) - \varepsilon, \forall\left(i, \theta, \theta'\right) \in I \times \Theta \times \Theta.$$

Definition 6 *For any $\varepsilon \geq 0$, a mechanism (q, m) defined on a belief subspace Θ satisfies ε-IM if*

$$\sup_{\theta \in \Theta} \rho\left(f\left(v\left(\theta\right)\right), q\left(\theta\right)\right) \leq \varepsilon.$$

For simplicity, we write SE, IC, and IM for 0-SE, 0-IC, and 0-IM, respectively. Observe that IM is the usual notion of exact implementation, i.e., $q\left(\theta\right) = f\left(v\left(\theta\right)\right)$ for every $\theta \in \Theta$. Moreover, when X is a finite set and ρ is the discrete metric, ε-IM with $\varepsilon < 1$ is equivalent to IM.

2.4 The space of all incomplete-information scenarios and genericity

Besides implementation of the SCF, the social planner may also aim to achieve one of the two goals: full surplus extraction (i.e., the Crémer-McLean (CM) setup) and budget balance (i.e., the Arrow-d'Aspremont-Gerard-Varet (AGV) setup). We now present the IC critique in both setups.

Define the space of all incomplete-information scenarios as

$$\mathcal{T} \equiv \left\{\Theta \subset \Theta^* : \Theta \text{ is a belief subspace}\right\}.$$

We endow $\mathcal{T}$ with the Hausdorff metric $d_{\mathcal{T}}$ such that

$$d_{\mathcal{T}}\left(\Theta, \Theta'\right) = \max\left\{\sup_{\theta \in \Theta} \inf_{\theta' \in \Theta'} d\left(\theta, \theta'\right), \sup_{\theta' \in \Theta'} \inf_{\theta \in \Theta} d\left(\theta, \theta'\right)\right\}, \forall \Theta, \Theta' \in \mathcal{T}.$$

Following Mertens and Zamir (1985), we say that $\Theta \in \mathcal{T}$ is a common-prior belief subspace if and only if there exists $\mu \in \Delta\left(\Theta^*\right)$ such that (1) supp$\mu = \Theta$, and (2) for every bounded measurable function $\varphi : \Theta^* \rightarrow \mathbb{R}$,

$$\int_{\Theta_i^*}\left(\int_{\Theta_{-i}^*} \varphi\left(\theta_i, \theta_{-i}\right) b_i\left(\theta_i\right)\left[d\theta_{-i}\right]\right) \mu_i\left[d\theta_i\right] = \int_{\Theta^*} \varphi\left(\theta\right) \mu\left[d\theta\right], \forall i, \tag{2}$$

where μ_i denotes the marginal distribution of μ on Θ_i^*. Let $\mathcal{P}$ denote the set of all common-prior incomplete-information scenarios;[3] more formally,

$$\mathcal{P} = \left\{\Theta \in \mathcal{T} : \Theta \text{ is a common-prior belief subspace}\right\}.$$

[3] An alternative approach to representing the space of all common-prior incomplete-information scenar-

We also endow $\mathcal{P}$ with the Hausdorff metric $d_{\mathcal{T}}$.

We focus on $\mathcal{T}$ when we present the IC critique in the AGV setup, and on $\mathcal{P}$ when we present the IC critique in the CM setup.[4]

Finally, we adopt the following standard genericity notion.[5]

Definition 7 *A set is generic if it contains an open and dense subset.*

3 The IC critique

To make the IC critique comparable to the FSE critique, we first present the IC critique in the CM setup, in which the designer aims to fully extract the surplus (i.e., to achieve SE). We then proceed to establish the IC critique in the AGV setup, in which the designer aims to balance the budget.

ios is to consider the set of all consistent priors on the universal type space, endowed with weak* topology (as in Chen and Xiong (2013a)). However, one technical difficulty for this approach is that no equilibrium may exist for complicated belief spaces, such as the universal type space (see, e.g., Hellman (2014), Simon (2003)). In Chen and Xiong (2013a), we consider the efficient SCF and the VCG mechanism, and hence an equilibrium always exists. In this paper, however, we consider an arbitrary SCF and arbitrary mechanisms, and therefore equilibrium existence may be problematic, unless we adopt a weaker (e.g., ex ante) IC notion.

[4]Revenue maximization becomes a problematic notion in belief subspaces without a common prior (see Börgers (2013)). A belief space with no common prior may admit a bet (i.e., a payment scheme) that is beneficial to all agents. As a result, a mechanism designer can use this bet to extract an arbitrarily large surplus without violating the IR condition.

[5]Our genericity notion (defined in terms of an open and dense subset) is stronger than the notion of residual sets used in Dekel, Fudenberg, and Morris (2006), Ely and Peski (2011) and Chen and Xiong (2011, 2013a). Our main results can be easily adjusted, however, if we use the notion of residual sets (see Chen and Xiong (2013a)).

3.1 The IC critique in the CM Setup

In the CM setup, we aim to achieve two goals, namely, implementation of the SCF and extraction of the agents' surplus. Define

$$\mathcal{P}^{\text{IR,SE,IM}} \equiv \{\Theta \in \mathcal{P} : \exists \ (q, m) \ \text{ on } \Theta \text{ which is IR, SE, and IM}\}.$$

That is, we ignore IC in defining $\mathcal{P}^{\text{IR,SE,IM}}$.

Proposition 1 $\mathcal{P}^{IR,SE,IM} = \mathcal{P}$.

Proposition 1 says that a mechanism design problem that does not require IC is trivial in the sense that f can *always* be implemented. Similarly, for any $\varepsilon > 0$, define

$$\begin{aligned} \mathcal{P}^{\text{IR},\varepsilon\text{-SE,IM}} &\equiv \{\Theta \in \mathcal{P} : \exists \ (q, m) \ \text{ on } \Theta \text{ which is IR, } \varepsilon\text{-SE and IM}\}; \\ \mathcal{P}^{\text{IR},\varepsilon\text{-SE},\varepsilon\text{-IM}} &\equiv \{\Theta \in \mathcal{P} : \exists \ (q, m) \ \text{ on } \Theta \text{ which is IR, } \varepsilon\text{-SE and } \varepsilon\text{-IM}\}. \end{aligned}$$

Here $\varepsilon > 0$ represents an arbitrarily small measurement error. Due to the technical difficulty involved with the general (infinite) priors, 0-SE may not be possible, and hence we follow McAfee and Reny (1992) in adopting ε-SE for any $\varepsilon > 0$, and similarly for ε-IM and ε-IC.

Clearly, Proposition 1 implies that

$$\mathcal{P}^{\text{IR,SE,IM}} = \mathcal{P}^{\text{IR},\varepsilon\text{-SE,IM}} = \mathcal{P}^{\text{IR},\varepsilon\text{-SE},\varepsilon\text{-IM}} = \mathcal{P}, \ \forall \varepsilon > 0.$$

The IC critique says that, generically, imposing IC does not make a difference. In particular, we consider two types of IC: the exact IC (i.e., 0-IC) and the "almost IC" (i.e., ε-IC). For any $\varepsilon > 0$, define

$$\begin{aligned} \mathcal{P}^{\text{IR},\varepsilon\text{-SE},\varepsilon\text{-IM,IC}} &\equiv \{\Theta \in \mathcal{T} : \exists \ (q, m) \ \text{ on } \Theta \text{ which is IR, } \varepsilon\text{-SE, } \varepsilon\text{-IM and IC}\}; \\ \mathcal{P}^{\text{IR},\varepsilon\text{-SE,IM},\varepsilon\text{-IC}} &\equiv \{\Theta \in \mathcal{T} : \exists \ (q, m) \ \text{ on } \Theta \text{ which is IR, } \varepsilon\text{-SE, IM and } \varepsilon\text{-IC}\}. \end{aligned}$$

Theorem 1 *For any $\varepsilon > 0$, both $\mathcal{P}^{IR,\varepsilon\text{-}SE,\varepsilon\text{-}IM,IC}$ and $\mathcal{P}^{IR,\varepsilon\text{-}SE,IM,\varepsilon\text{-}IC}$ are generic in $\mathcal{P}$.*

Theorem 1 implies the following:

$$\mathcal{P}^{\text{IR},\varepsilon\text{-SE},\varepsilon\text{-IM,IC}} \text{ is generic in } \mathcal{P}^{\text{IR},\varepsilon\text{-SE},\varepsilon\text{-IM}} = \mathcal{P};$$
$$\mathcal{P}^{\text{IR},\varepsilon\text{-SE,IM},\varepsilon\text{-IC}} \text{ is generic in } \mathcal{P}^{\text{IR},\varepsilon\text{-SE,IM}} = \mathcal{P}.$$

In other words, generically, there is no difference between imposing and not imposing IC. Thus, we obtain the IC critique in the CM setup.

Note that in Theorem 1 we get a weaker IM condition if we require a stronger IC, and we get a stronger IM condition if we require a weaker IC. Nevertheless, the weaker IM and IC can be arbitrarily close to the stronger IM and IC, respectively, i.e., ε can be arbitrarily small.

3.2 The IC critique in the AGV setup

In the AGV setup, we aim to achieve two goals, namely, implementation of the SCF and budget balance. Define

$$\mathcal{T}^{\text{IM,BB}} \equiv \{\Theta \in \mathcal{T} : \exists\ (q, m) \text{ on } \Theta \text{ which is IM and BB}\}.$$

That is, we ignore IC in defining $\mathcal{T}^{\text{IM,BB}}$.

Proposition 2 $\mathcal{T}^{IM,BB} = \mathcal{T}$.

Proposition 2 says that a mechanism design problem that does not require IC is trivial in the sense that f can *always* be implemented. Similarly, for any $\varepsilon > 0$, define

$$\mathcal{T}^{\varepsilon\text{-IM,BB}} \equiv \{\Theta \in \mathcal{T} : \exists\ (q, m) \text{ on } \Theta \text{ which is } \varepsilon\text{-IM and BB}\}.$$

Clearly, Proposition 2 implies that

$$\mathcal{T}^{\text{IM,BB}} = \mathcal{T}^{\varepsilon\text{-IM,BB}} = \mathcal{T},\ \forall \varepsilon > 0.$$

The IC critique says that, generically, imposing IC does not make a difference. In particular, we consider two types of IC: the exact IC (i.e., 0-IC) and the "almost IC" (i.e., ε-IC).

For any $\varepsilon > 0$, define

$$\begin{aligned}
\mathcal{T}^{\varepsilon\text{-IM,BB,IC}} &\equiv \{\Theta \in \mathcal{T} : \exists\ (q, m) \text{ on } \Theta \text{ which is } \varepsilon\text{-IM, BB and IC}\}\,; \\
\mathcal{T}^{\text{IM,BB},\varepsilon\text{-IC}} &\equiv \{\Theta \in \mathcal{T} : \exists\ (q, m) \text{ on } \Theta \text{ which is IM, BB and } \varepsilon\text{-IC}\}\,.
\end{aligned}$$

Theorem 2 *If $|I| \geq 3$, then for any $\varepsilon > 0$, both $\mathcal{T}^{\varepsilon\text{-IM,BB,IC}}$ and $\mathcal{T}^{\text{IM,BB},\varepsilon\text{-IC}}$ are generic in $\mathcal{T}$.*

Theorem 2 implies that

$$\begin{aligned}
\mathcal{T}^{\varepsilon\text{-IM,BB,IC}} \text{ is generic in } \mathcal{T}^{\varepsilon\text{-IM,BB}} &= \mathcal{T}; \\
\mathcal{T}^{\text{IM,BB},\varepsilon\text{-IC}} \text{ is generic in } \mathcal{T}^{\text{IM,BB}} &= \mathcal{T}.
\end{aligned}$$

That is, generically, there is no difference between imposing and not imposing IC. Thus, we obtain the IC critique in the AGV setup.

4 Conclusion

Economic models are parsimonious ways to describe an often complicated reality. As a result, many subtle, albeit important, details may be assumed away. For instance, even though the classical mechanism design model assumes away risk aversion, limited liability, collusion among agents, and competition among sellers (see Robert (1991), Laffont and Martimort (2000), Che and Kim (2006), and Peters (2001)), the model is still adopted extensively in the economics literature, due to its analytical tractability.

In this paper, instead of studying surplus extraction, we have focused on the most fundamental requirement of mechanism design models: incentive compatibility. We prove that a designer facing a generic incomplete-information scenario can dispense with the IC constraint without incurring any loss of generality. This result suggests that when economists adopt the classical mechanism design model, they implicitly take the position that asymmetric information does not have bites generically. Consequently, this "cast(s) doubt on the value of the current mechanism design paradigm as a model of institutional design" (McAfee and Reny, 1992, p. 400). Therefore, our result demonstrates the need for a new paradigm for mechanism design.

A Proofs

A.1 The proof of Proposition 1

Proposition 1. $\mathcal{P}^{\text{IR,SE,IM}} = \mathcal{P}$.

Proof. Define a mechanism (q, m) on Θ^* as follows.

$$q(\theta) = f(v(\theta)) \text{ and } m_i(\theta) = u_i(v(\theta), f(v(\theta))), \forall (i, \theta) \in I \times \Theta^*.$$

It is easy to check $u_i(\theta_i, \theta_{-i}|\theta_i, q, m) \equiv 0$. As a result, $U_i(\theta_i|q, m) = 0$ for every θ_i. Hence, (q, m) satisfies IR, SE and IM. ■

A.2 The proof of Proposition 2

Proposition 2. $\mathcal{T}^{\text{IM,BB}} = \mathcal{T}$.

Proof. Define a mechanism (q, m) on Θ^* as follows.

$$q(\theta) = f(v(\theta)) \text{ and } m_i(\theta) = 0, \forall (i, \theta) \in I \times \Theta^*.$$

Clearly, (q, m) satisfies IM and BB. ■

A.3 The proof of Theorem 1

Theorem 1. *For any $\varepsilon > 0$, both $\mathcal{P}^{\text{IR},\varepsilon\text{-SE},\varepsilon\text{-IM,IC}}$ and $\mathcal{P}^{\text{IR},\varepsilon\text{-SE,IM},\varepsilon\text{-IC}}$ are generic in $\mathcal{P}$.*

Proof. Like Chen and Xiong (2013a), we define "full rank" as follows. For every $\Theta \in \mathcal{P}$ with $|\Theta| < \infty$, define

$$\mathbf{B}_i^{\Theta} \equiv [b_i(\theta_i)[\theta_{-i}]]_{\theta_i \in \Theta_i, \theta_{-i} \in \Theta_{-i}},$$

where each row corresponds to a type θ_i of agent i and his belief about his opponents' types, i.e., $[b_i(\theta_i)[\theta_{-i}]]_{\theta_{-i} \in \Theta_{-i}}$. We say $\mathbf{B}_i^{\Theta}$ has *full rank* if its column rank is equal to $|\Theta_i|$.

Say Θ has full rank if $\mathbf{B}_i^{\Theta}$ has full rank for every $i \in I$. By (Chen and Xiong, 2013b, Lemmas S.2 and S.3), $\mathcal{P}^{\mathrm{CM}}$ defined below is dense in $\mathcal{P}$.

$$\mathcal{P}^{\mathrm{CM}} \equiv \left\{\Theta \in \mathcal{P} : \ |\Theta| < \infty, \mathbf{B}_i^{\Theta} \text{ has full rank}\right\}. \tag{3}$$

Assumption 1 further implies that $\mathcal{P}^{\mathrm{CM}^*}$ defined below is dense in $\mathcal{P}$.

$$\mathcal{P}^{\mathrm{CM}^*} \equiv \left\{\Theta \in \mathcal{P} : \begin{array}{c} |\Theta| < \infty, \mathbf{B}_i^{\Theta} \text{ has full rank} \\ \text{and } f\left(v\left(\theta\right)\right) \text{ is continuous at } \theta, \forall \theta \in \Theta \end{array}\right\}.$$

The following two lemmas complete the proof of Theorem 1.

Lemma 1 *For any $\varepsilon > 0$ and every $\Theta \in \mathcal{P}^{CM^*}$, there exists $\delta > 0$ such that*

$$\left\{\widehat{\Theta} \in \mathcal{P} : d_{\mathcal{T}}\left(\Theta, \widehat{\Theta}\right) < \delta\right\} \subset \mathcal{P}^{IR,\varepsilon\text{-}SE,\varepsilon\text{-}IM,IC}.$$

Lemma 2 *For any $\varepsilon > 0$ and every $\Theta \in \mathcal{P}^{CM^*}$, there exists $\delta > 0$ such that*

$$\left\{\widehat{\Theta} \in \mathcal{P} : d_{\mathcal{T}}\left(\Theta, \widehat{\Theta}\right) < \delta\right\} \subset \mathcal{P}^{IR,\varepsilon\text{-}SE,IM,\varepsilon\text{-}IC}.$$

By Lemmas 1 and 2, for any $\varepsilon > 0$ and every $\Theta \in \mathcal{P}^{\mathrm{CM}^*}$, there exists $\delta^{\Theta} > 0$ such that

$$\left\{\widehat{\Theta} \in \mathcal{P} : d_{\mathcal{T}}\left(\Theta, \widehat{\Theta}\right) < \delta^{\Theta}\right\} \subset \mathcal{P}^{\mathrm{IR},\varepsilon\text{-SE},\varepsilon\text{-IM,IC}} \cap \mathcal{P}^{\mathrm{IR},\varepsilon\text{-SE,IM},\varepsilon\text{-IC}}.$$

As a result,

$$\bigcup_{\Theta \in \mathcal{P}^{\mathrm{CM}^*}} \left\{\widehat{\Theta} \in \mathcal{P} : d_{\mathcal{T}}\left(\Theta, \widehat{\Theta}\right) < \delta^{\Theta}\right\} \subset \mathcal{P}^{\mathrm{IR},\varepsilon\text{-SE},\varepsilon\text{-IM,IC}} \cap \mathcal{P}^{\mathrm{IR},\varepsilon\text{-SE,IM},\varepsilon\text{-IC}}.$$

Since $\mathcal{P}^{\mathrm{CM}^*}$ is dense in $\mathcal{P}$, $\bigcup_{\Theta \in \mathcal{P}^{\mathrm{CM}^*}} \left\{\widehat{\Theta} \in \mathcal{P} : d_{\mathcal{T}}\left(\Theta, \widehat{\Theta}\right) < \delta^{\Theta}\right\}$ is open and dense in $\mathcal{P}$. Therefore, both $\mathcal{P}^{\mathrm{IR},\varepsilon\text{-SE},\varepsilon\text{-IM,IC}}$ and $\mathcal{P}^{\mathrm{IR},\varepsilon\text{-SE,IM},\varepsilon\text{-IC}}$ are generic in $\mathcal{P}$. ■

A.3.1 Proof of Lemmas 1 and 2

In order to prove Lemmas 1 and 2, we first prove a preliminary result.

Lemma 3 *For every $\Theta \in \mathcal{P}^{CM*}$, there exists a mechanism (q, m) defined on Θ such that*

$$q(\theta) \equiv f(v(\theta)) \text{ and } \tag{4}$$

$$0 = U_i(\theta_i | q, m) > U_i(\theta_i' | \theta_i, q, m), \forall \theta_i \neq \theta_i' \text{ and } \theta_i, \theta_i' \in \Theta_i \tag{5}$$

That is, (q, m) satisfies IR, SE, IM, and IC.

Proof. First, since Θ has full rank, $b_i(\theta_i) \neq b_i(\theta_i')$ for every distinct pair $\theta_i, \theta_i' \in \Theta_i$. For every positive integer $K > 0$, consider a mechanism (q, m^K) on Θ defined as follows.

$$q(\theta) \equiv f(v(\theta));$$

$$m_i^K(\theta) \equiv K \left[2b_i(\theta_i)[\theta_{-i}] - \sum_{\theta'_{-i} \in \Theta_{-i}} \left(b_i(\theta_i)[\theta'_{-i}] \right)^2 \right].$$

That is, $m_i^K(\cdot)$ is a K multiple of the quadratic proper scoring rule. Since $b_i(\theta_i) \neq b_i(\theta_i')$, it follows that

$$\sum_{\theta_{-i}} b_i(\theta_i)[\theta_{-i}] m_i^K(\theta_i, \theta_{-i}) - \sum_{\theta_{-i}} b_i(\theta_i)[\theta_{-i}] m_i^K(\theta_i', \theta_{-i}) > 0, \forall \theta_i \neq \theta_i'. \tag{6}$$

Moreover, the difference in (6) can be made arbitrarily large by increasing K. Since $U_i(\theta_i' | \theta_i, q, m^0 \equiv 0)$ is bounded for all θ_i, θ_i', we can choose sufficiently large K so that

$$U_i\left(\theta_i | q, m^K\right) > U_i\left(\theta_i' | \theta_i, q, m^K\right), \forall \theta_i \neq \theta_i'.$$

Finally, for every i, since $\mathbf{B}_i^\Theta \equiv [b_i(\theta_i)[\theta_{-i}]]_{\theta_i \in \Theta_i, \theta_{-i} \in \Theta_{-i}}$ has full rank, we can find a column vector $[w_i(\theta_{-i})]_{\theta_{-i} \in \Theta_{-i}}$ such that the following matrix equation holds:

$$[b_i(\theta_i)[\theta_{-i}]]_{\theta_i \in \Theta_i, \theta_{-i} \in \Theta_{-i}} \times [w_i(\theta_{-i})]_{\theta_{-i} \in \Theta_{-i}} = \left[U_i\left(\theta_i | q, m^K\right) \right]_{\theta_i \in \Theta_i}.$$

That is, there is some side payment scheme $(w_i : \Theta_{-i} \to \mathbb{R})_{i \in I}$ such that

$$\sum_{\theta_{-i}} b_i(\theta_i)[\theta_{-i}] w_i(\theta_{-i}) = U_i\left(\theta_i | q, m^K\right), \forall \theta_i \in \Theta_i.$$

It follows that (q, m) with $m_i = m_i^K + w_i$ is the desired mechanism.■

We now prove Lemmas 1 and 2. Fix any $\Theta \in \mathcal{P}^{CM*}$. Let (q, m) denote the mechanism we obtain from Lemma 3. Define

$$\zeta \equiv \min_{i \in I} \min_{\theta_i, \theta_i' \in \Theta_i, \theta_i \neq \theta_i'} \left| U_i(\theta_i | q, m) - U_i(\theta_i' | \theta_i, q, m) \right| > 0. \tag{7}$$

$$\vartheta \equiv \min_{i\in I} \min_{\theta_i,\theta_i'\in\Theta_i,\,\theta_i\neq\theta_i'} d_i\left(\theta_i,\theta_i'\right) > 0. \tag{8}$$

First, define $q^* : \Theta^* \to X$ as $q^*(\theta) = f(v(\theta))$ for every $\theta \in \Theta^*$. Second, define $\Theta^{\frac{\vartheta}{4}} \equiv \left\{\widehat{\theta} \in \Theta^* : \exists \theta \in \Theta \text{ s.t. } d\left(\theta,\widehat{\theta}\right) \le \frac{\vartheta}{4}\right\}$. By the definition of ϑ in (8), for each $\widehat{\theta} \in \Theta^{\frac{\vartheta}{4}}$, there is exactly one $\theta \in \Theta$ such that $d\left(\theta,\widehat{\theta}\right) \le \frac{\vartheta}{4}$. Consider the functions $q^{**} : \Theta^{\frac{\vartheta}{4}} \to X$ and $\left(m_i^{**} : \Theta^{\frac{\vartheta}{4}} \to \mathbb{R}\right)_{i\in I}$ such that for any $\left(\theta,\widehat{\theta}\right) \in \Theta \times \Theta^{\frac{\vartheta}{4}}$,

$$d\left(\theta,\widehat{\theta}\right) \le \frac{\vartheta}{4} \Longrightarrow q^{**}\left(\widehat{\theta}\right) = q(\theta) \text{ and } m_i^{**}\left(\widehat{\theta}\right) = m_i(\theta), \forall i \in I. \tag{9}$$

Proof of Lemma 1 Recall that $f(v(\theta))$ is continuous at every $\theta \in \Theta$ and $|\Theta| < \infty$. Since q^{**} and $(m_i^{**})_{i\in I}$ are continuous on $\Theta^{\frac{\vartheta}{4}}$, there exists $\delta^{**} \in \left(0, \frac{\vartheta}{4}\right)$ such that for any $\widehat{\Theta} \in \mathcal{P}$ with $d_{\mathcal{T}}\left(\Theta,\widehat{\Theta}\right) < \delta^{**}$ and any $\left(\theta,\theta',\widehat{\theta},\widehat{\theta}'\right) \in \Theta\times\Theta\times\widehat{\Theta}\times\widehat{\Theta}$, we have

$$\max\left\{d\left(\theta,\widehat{\theta}\right), d\left(\theta',\widehat{\theta}'\right)\right\} < \delta^{**} \Longrightarrow \tag{10}$$
$$\left\{\begin{array}{l} \rho\left(f(v(\theta)), f\left(v\left(\widehat{\theta}\right)\right)\right) < \varepsilon \text{ and} \\ \left|U_i\left(\theta_i'|\theta_i, q^{**}, m^{**}\right) - U_i\left(\widehat{\theta}_i'|\widehat{\theta}_i, q^{**}, m^{**}\right)\right| < \min\left\{\frac{\zeta}{4}, \frac{\varepsilon}{4}\right\}. \end{array}\right.$$

We now prove

$$\left\{\widehat{\Theta} \in \mathcal{P} : d_{\mathcal{T}}\left(\Theta,\widehat{\Theta}\right) < \delta^{**}\right\} \subset \mathcal{P}^{\text{IR},\varepsilon\text{-SE},\varepsilon\text{-IM,IC}}.$$

Fix any $\widehat{\Theta} \in \mathcal{P}$ such that $d_{\mathcal{T}}\left(\Theta,\widehat{\Theta}\right) < \delta^{**}$, and we show $\widehat{\Theta} \in \mathcal{P}^{\text{IR},\varepsilon\text{-SE},\varepsilon\text{-IM,IC}}$. For any $\widehat{\theta} \in \widehat{\Theta}$, since $\delta^{**} < \frac{\vartheta}{4}$, let θ denote the unique type in Θ such that $d\left(\theta,\widehat{\theta}\right) < \frac{\vartheta}{4}$.

First, by (4), (9) and (10), we have

$$\rho\left(q^{**}\left(\widehat{\theta}\right), f\left(v\left(\widehat{\theta}\right)\right)\right) = \rho\left(q(\theta), f\left(v\left(\widehat{\theta}\right)\right)\right) = \rho\left(f(v(\theta)), f\left(v\left(\widehat{\theta}\right)\right)\right) < \varepsilon.$$

That is, (q^{**}, m^{**}) on $\widehat{\Theta}$ satisfies ε-IM.

Second, we show that (q^{**}, m^{**}) on $\widehat{\Theta}$ satisfies $\frac{\varepsilon}{4}$-SE. Note that

$$\begin{aligned} & \left|U_i\left(\theta_i|q^{**}, m^{**}\right) - U_i\left(\widehat{\theta}_i|q^{**}, m^{**}\right)\right| \\ = & \left|U_i\left(\theta_i|\theta_i, q^{**}, m^{**}\right) - U_i\left(\widehat{\theta}_i|\widehat{\theta}_i, q^{**}, m^{**}\right)\right| \\ < & \min\left\{\frac{\zeta}{4}, \frac{\varepsilon}{4}\right\}, \end{aligned} \tag{11}$$

where the inequality follows from (10). Furthermore, (9) implies $U_i(\theta_i|q^{**}, m^{**}) = U_i(\theta_i|q, m) = 0$. Hence, (11) implies

$$\left|U_i\left(\widehat{\theta}_i|q^{**}, m^{**}\right)\right| < \min\left\{\frac{\zeta}{4}, \frac{\varepsilon}{4}\right\}. \tag{12}$$

That is, (q^{**}, m^{**}) on $\widehat{\Theta}$ satisfies $\frac{\varepsilon}{4}$-SE.

Third, we show that IC is satisfied for (q^{**}, m^{**}) defined on $\widehat{\Theta}$. If type $\widehat{\theta}_i \in \widehat{\Theta}_i$ deviates to report some other $\widehat{\theta}'_i \in \widehat{\Theta}_i$, with θ'_i being the unique type in Θ_i such that $d_i\left(\widehat{\theta}'_i, \theta'_i\right) < \frac{\vartheta}{4}$. Consider two cases: i) $\theta_i = \theta'_i$ and ii) $\theta_i \neq \theta'_i$. In case i), (9) implies

$$U_i\left(\widehat{\theta}_i|q^{**}, m^{**}\right) = U_i\left(\theta_i|\widehat{\theta}_i, q^{**}, m^{**}\right) = U_i\left(\theta'_i|\widehat{\theta}_i, q^{**}, m^{**}\right) = U_i\left(\widehat{\theta}'_i|\widehat{\theta}_i, q^{**}, m^{**}\right).$$

In case ii), we have

$$\begin{aligned} U_i\left(\widehat{\theta}_i|q^{**}, m^{**}\right) &> U_i(\theta_i|q^{**}, m^{**}) - \frac{\zeta}{4} \\ &= U_i(\theta_i|q, m) - \frac{\zeta}{4} \\ &> U_i(\theta'_i|\theta_i, q, m) + \frac{\zeta}{4} \\ &= U_i(\theta'_i|\theta_i, q^{**}, m^{**}) + \frac{\zeta}{4} \\ &> U_i\left(\widehat{\theta}'_i|\widehat{\theta}_i, q^{**}, m^{**}\right). \end{aligned}$$

where the first inequality follows from (11); the two equalities follow from (9); the second inequality follows from (5) and (7); the last inequality follows from (10). Thus, in both cases, reporting the true type is optimal, and IC is satisfied.

Finally, we take care of IR. Note that (12) implies that

$$-\frac{\varepsilon}{4} < U_i\left(\widehat{\theta}_i|\widehat{\theta}_i, q^{**}, m^{**}\right) < \frac{\varepsilon}{4}. \tag{13}$$

If we give a lump sum of $\frac{\varepsilon}{4}$ to every type of every agent, IR would be satisfied; meanwhile, IC and ε-IM would remain unchanged. Define $\left(m'_i : \Theta^* \to \mathbb{R}\right)_{i\in I}$ as

$$m'_i(\theta) \equiv m^{**}_i(\theta) - \frac{\varepsilon}{4}. \tag{14}$$

Then, (13) and (14) implies

$$0 < U_i\left(\widehat{\theta}_i|q^{**}, m'\right) < \frac{\varepsilon}{2}, \forall \widehat{\theta}_i \in \widehat{\Theta}_i.$$

Therefore, (q^{**}, m') on $\widehat{\Theta}$ satisfies IR, ε-SE, ε-IM, and IC.■

Proof of Lemma 2 Recall that $f(v(\theta))$ is continuous at every $\theta \in \Theta$ and $|\Theta| < \infty$. Hence, q^* and $(m_i^{**})_{i \in I}$ are continuous at every $\theta \in \Theta$. It follows that there exists $\delta^* \in \left(0, \frac{\vartheta}{4}\right)$ such that for any any $\widehat{\Theta} \in \mathcal{P}$ with $d_{\mathcal{T}}\left(\Theta, \widehat{\Theta}\right) < \delta^*$ and any $\left(\theta, \theta', \widehat{\theta}, \widehat{\theta}'\right) \in \Theta \times \Theta \times \widehat{\Theta} \times \widehat{\Theta}$, we have

$$\max\left\{d\left(\theta, \widehat{\theta}\right), d\left(\theta', \widehat{\theta}'\right)\right\} < \delta^* \implies \left|U_i\left(\theta_i'|\theta_i, q^*, m^{**}\right) - U_i\left(\widehat{\theta}_i'|\widehat{\theta}_i, q^*, m^{**}\right)\right| < \frac{\varepsilon}{4}. \quad (15)$$

We now prove

$$\left\{\widehat{\Theta} \in \mathcal{P} : d_{\mathcal{T}}\left(\Theta, \widehat{\Theta}\right) < \delta^*\right\} \subset \mathcal{P}^{\text{IR},\varepsilon\text{-SE},\text{IM},\varepsilon\text{-IC}}.$$

Fix any $\widehat{\Theta} \in \mathcal{P}$ such that $d_{\mathcal{T}}\left(\Theta, \widehat{\Theta}\right) < \delta^*$. Since $\delta^* < \frac{\vartheta}{4}$, for any $\widehat{\theta}_i, \widehat{\theta}_i' \in \widehat{\Theta}_i$, let θ_i, θ_i' denote the unique types in Θ_i such that $\max\left\{d_i\left(\widehat{\theta}_i, \theta_i\right), d_i\left(\widehat{\theta}_i', \theta_i'\right)\right\} < \delta^*$.

First, recall that $q^*(\theta) = f(v(\theta))$ for every $\theta \in \Theta^*$. Hence, (q^*, m^{**}) on $\widehat{\Theta}$ satisfies IM.

Second, (15) implies that

$$\left|U_i\left(\theta_i|q^*, m^{**}\right) - U_i\left(\widehat{\theta}_i|q^*, m^{**}\right)\right| < \frac{\varepsilon}{4}. \quad (16)$$

Note that (q^*, m^{**}) coincides with (q, m) on Θ, and hence

$$U_i\left(\theta_i|q^*, m^{**}\right) = U_i\left(\theta_i|q, m\right) = 0, \quad (17)$$

where the last equality follows from (5). Thus, (16) and (17) imply

$$\left|U_i\left(\widehat{\theta}_i|q^*, m^{**}\right)\right| < \frac{\varepsilon}{4}. \quad (18)$$

That is, (q^*, m^{**}) on $\widehat{\Theta}$ satisfies $\frac{\varepsilon}{4}$-SE.

Third, we show that (q^*, m^{**}) on $\widehat{\Theta}$ satisfies ε-IC. Since (q^*, m^{**}) is an extension of (q, m) on Θ, for any distinct types θ_i and θ_i' in Θ_i, we have

$$U_i\left(\theta_i|q^*, m^{**}\right) - U_i\left(\theta_i'|\theta_i, q^*, m^{**}\right) = U_i\left(\theta_i|q, m\right) - U_i\left(\theta_i'|\theta_i, q, m\right) > 0, \quad (19)$$

where the inequality follows from (5). Hence, (15) and (19) imply

$$U_i\left(\widehat{\theta}_i|q^*, m^{**}\right) - U_i\left(\widehat{\theta}_i'|\widehat{\theta}_i, q^*, m^{**}\right) \geq -2 \times \frac{\varepsilon}{4} > -\varepsilon.$$

Thus, reporting the true type is an ε-best reply. Therefore, (q^*, m^{**}) on $\widehat{\Theta}$ satisfies ε-IC.

Finally, we take care of IR. Note that (18) implies that

$$-\frac{\varepsilon}{4} < U_i\left(\widehat{\theta}_i | q^*, m^{**}\right) < \frac{\varepsilon}{4}. \tag{20}$$

Recall that $m_i'(\theta) \equiv m_i^{**}(\theta) - \frac{\varepsilon}{4}$. Thus, (q^*, m') on $\widehat{\Theta}$ satisfies IR, ε-SE, IM, and ε-IC.■

A.4 The proof of Theorem 2

Theorem 2. *For any $\varepsilon > 0$, if $|I| \geq 3$, both $\mathcal{T}^{\varepsilon\text{-IM,BB,IC}}$ and $\mathcal{T}^{\text{IM,BB},\varepsilon\text{-IC}}$ are generic in $\mathcal{T}$.*

Proof. Suppose $|I| \geq 3$ and $|\Theta| < \infty$. We define "full support" and "distinct beliefs" as follows. We say $\Theta \in \mathcal{T}$ has full support iff for every $(i, j, \theta) \in I \times I \times \Theta$,

$$b_i(\theta_i)\left[\theta_{-i-j}\right] > 0.$$

We say $\Theta \in \mathcal{T}$ has distinct beliefs iff for every $(i, j, \theta, \theta') \in I \times I \times \Theta \times \Theta$,

$$\theta_i \neq \theta_i' \implies b_i(\theta_i)\left[\theta_{-i-j}\right] \neq b_i(\theta_i')\left[\theta_{-i-j}\right].$$

It is easy to see that the set $\mathcal{T}^{\text{AGV}}$ defined below is dense in $\mathcal{T}$.

$$\mathcal{T}^{\text{AGV}} \equiv \left\{\Theta \in \mathcal{T} : \begin{array}{c} |\Theta| < \infty, \\ \Theta \text{ has full support and distinct beliefs} \end{array}\right\}.$$

Assumption 1 further implies that $\mathcal{T}^{\text{AGV*}}$ defined below is dense in $\mathcal{T}$.

$$\mathcal{T}^{\text{AGV*}} \equiv \left\{\Theta \in \mathcal{T} : \begin{array}{c} |\Theta| < \infty, \\ \Theta \text{ has full support and distinct beliefs} \\ \text{and } f(v(\theta)) \text{ is continuous at } \theta, \forall \theta \in \Theta \end{array}\right\}.$$

The following two lemmas complete the proof of Theorem 2.

Lemma 4 *If $|I| \geq 3$, for any $\varepsilon > 0$ and every $\Theta \in \mathcal{T}^{AGV*}$, there exists $\delta > 0$ such that*

$$\left\{\widehat{\Theta} \in \mathcal{T} : d_{\mathcal{T}}\left(\Theta, \widehat{\Theta}\right) < \delta\right\} \subset \mathcal{T}^{\varepsilon\text{-IM,BB,IC}}.$$

Lemma 5 *If $|I| \geq 3$, for any $\varepsilon > 0$ and every $\Theta \in \mathcal{T}^{AGV*}$, there exists $\delta > 0$ such that*

$$\left\{\widehat{\Theta} \in \mathcal{T} : d_{\mathcal{T}}\left(\Theta, \widehat{\Theta}\right) < \delta\right\} \subset \mathcal{T}^{IM,BB,\varepsilon\text{-}IC}.$$

By Lemmas 4 and 5, for any $\varepsilon > 0$ and every $\Theta \in \mathcal{T}^{\text{AGV*}}$, there exists $\delta^{\Theta} > 0$ such that

$$\left\{\widehat{\Theta} \in \mathcal{P} : d_{\mathcal{T}}\left(\Theta, \widehat{\Theta}\right) < \delta^{\Theta}\right\} \subset \mathcal{T}^{\varepsilon\text{-IM,BB,IC}} \cap \mathcal{T}^{\text{IM,BB},\varepsilon\text{-IC}}.$$

As a result,

$$\bigcup_{\Theta \in \mathcal{T}^{\text{AGV*}}} \left\{\widehat{\Theta} \in \mathcal{P} : d_{\mathcal{T}}\left(\Theta, \widehat{\Theta}\right) < \delta^{\Theta}\right\} \subset \mathcal{T}^{\varepsilon\text{-IM,BB,IC}} \cap \mathcal{T}^{\text{IM,BB},\varepsilon\text{-IC}}.$$

Since $\mathcal{T}^{\text{AGV*}}$ is dense in $\mathcal{T}$, $\bigcup_{\Theta \in \mathcal{T}^{\text{AGV*}}} \left\{\Theta' \in \mathcal{P} : d_{\mathcal{T}}\left(\Theta, \widehat{\Theta}\right) < \delta^{\Theta}\right\}$ is open and dense in $\mathcal{T}$. Therefore, both $\mathcal{T}^{\varepsilon\text{-IM,BB,IC}}$ and $\mathcal{T}^{\text{IM,BB},\varepsilon\text{-IC}}$ are generic in $\mathcal{T}$. ■

A.4.1 Proof of Lemmas 4 and 5

In order to prove Lemmas 4 and 5, we first prove a preliminary result.

Lemma 6 *For every $\Theta \in \mathcal{T}^{AGV*}$, there exists a mechanism (q, m) defined on Θ such that*

$$q(\theta) = f(v(\theta)), \forall \theta \in \Theta; \tag{21}$$

$$\sum_{i \in I} m_i(\theta) = 0, \forall \theta \in \Theta; \tag{22}$$

$$U_i(\theta_i | q, m) > U_i(\theta_i' | \theta_i, q, m), \forall \theta_i \neq \theta_i'. \tag{23}$$

That is, (q, m) satisfies IM, BB and IC.

Proof. The argument is standard (see, e.g., d'Aspremont, Crémer, and Gérard-Varet (2003)): list the agents as $1, 2, ..., I$; for every positive integer $K > 0$, consider a mechanism (q, m^K) on Θ defined as follows.

$$q(\theta) \equiv f(v(\theta));$$

$$m_i^K(\theta) \equiv K \times \begin{pmatrix} \log b_i(\theta_i)\left[\theta_{-i-(i-1)}\right] - \log b_{(i-1)}\left(\theta_{(i-1)}\right)\left[\theta_{-i-(i-1)}\right] \\ + \log b_i(\theta_i)\left[\theta_{-i-(i+1)}\right] - \log b_{(i+1)}\left(\theta_{(i+1)}\right)\left[\theta_{-i-(i+1)}\right] \end{pmatrix},$$

where, for notational ease, agents 0 and $I+1$ are interpreted as agents I and 1, respectively. It is easy to check $\sum_{i\in I} m_i^K(\theta) \equiv 0$. Furthermore, since m_i^K is a K multiple of the logarithmic scoring rule, we have

$$\sum_{\theta_{-i}} b_i(\theta_i)[\theta_{-i}]\, m_i^K(\theta_i,\theta_{-i}) - \sum_{\theta_{-i}} b_i(\theta_i)[\theta_{-i}]\, m_i^K(\theta_i',\theta_{-i}) > 0, \forall \theta_i \neq \theta_i'. \tag{24}$$

Moreover, the difference in (24) can be made arbitrarily large by increasing K. Since $U_i(\theta_i'|\theta_i, q, m^0 \equiv 0)$ is bounded for all θ_i, θ_i', we can choose sufficiently large K so that

$$U_i\left(\theta_i|q,m^K\right) > U_i\left(\theta_i'|\theta_i,q,m^K\right), \forall \theta_i \neq \theta_i'.$$

Hence, (23) holds.■

We now prove Lemmas 4 and 5. Fix any $\Theta \in \mathcal{T}^{\text{AGV*}}$. Let (q,m) denote the mechanism we obtain from Lemma 6. Define

$$\zeta \equiv \min_{i\in I} \min_{\theta_i,\theta_i'\in\Theta_i,\, \theta_i\neq\theta_i'} \left|U_i(\theta_i|q,m) - U_i(\theta_i'|\theta_i,q,m)\right| > 0. \tag{25}$$

$$\vartheta \equiv \min_{i\in I} \min_{\theta_i,\theta_i'\in\Theta_i,\, \theta_i\neq\theta_i'} d_i(\theta_i,\theta_i') > 0. \tag{26}$$

First, define $q^*:\Theta^* \to X$ as $q^*(\theta) = f(v(\theta))$ for every $\theta \in \Theta^*$. Second, because of the definition of ϑ in (26), for each $\widehat{\theta} \in \Theta^*$, there is at most one $\theta \in \Theta$ such that $d\left(\theta,\widehat{\theta}\right) \leq \frac{\vartheta}{4}$. Hence, as above, we can extend $q:\Theta \to X$ and $(m_i:\Theta \to \mathbb{R})_{i\in I}$ to bounded and continuous functions $q^{**}:\Theta^{\frac{\vartheta}{4}} \to X$ and $\left(m_i^{**}:\Theta^{\frac{\vartheta}{4}} \to \mathbb{R}\right)_{i\in I}$ such that for any $\left(\theta,\widehat{\theta}\right) \in \Theta \times \Theta^{\frac{\vartheta}{4}}$,

$$d\left(\theta,\widehat{\theta}\right) \leq \frac{\vartheta}{4} \Longrightarrow q^{**}\left(\widehat{\theta}\right) = q(\theta) \text{ and } m_i^{**}\left(\widehat{\theta}\right) = m_i(\theta), \forall i \in I. \tag{27}$$

Proof of Lemma 4 Recall that $f(v(\theta))$ is continuous at every $\theta \in \Theta$ and $|\Theta| < \infty$. Since q^{**} and $(m_i^{**})_{i\in I}$ are continuous on $\Theta^{\frac{\vartheta}{4}}$, there exists $\delta^{**} \in \left(0,\frac{\vartheta}{4}\right)$ such that for any any $\widehat{\Theta} \in \mathcal{P}$ with $d_{\mathcal{T}}\left(\Theta,\widehat{\Theta}\right) < \delta^{**}$ and any $\left(\theta,\theta',\widehat{\theta},\widehat{\theta}'\right) \in \Theta \times \Theta \times \widehat{\Theta} \times \widehat{\Theta}$, we have

$$\max\left\{d\left(\theta,\widehat{\theta}\right), d\left(\theta',\widehat{\theta}'\right)\right\} < \delta^{**} \Longrightarrow \tag{28}$$
$$\begin{cases} \rho\left(f(v(\theta)), f\left(v\left(\widehat{\theta}\right)\right)\right) < \varepsilon \text{ and} \\ \left|U_i(\theta_i'|\theta_i,q^{**},m^{**}) - U_i\left(\widehat{\theta}_i'|\widehat{\theta}_i,q^{**},m^{**}\right)\right| < \min\left\{\frac{\zeta}{4},\frac{\varepsilon}{4}\right\}. \end{cases}$$

We now prove

$$\left\{\widehat{\Theta} \in \mathcal{T} : d_{\mathcal{T}}\left(\Theta, \widehat{\Theta}\right) < \delta^{**}\right\} \subset \mathcal{T}^{\varepsilon\text{-IM,BB,IC}}.$$

Fix any $\widehat{\Theta} \in \mathcal{T}$ such that $d_{\mathcal{T}}\left(\Theta, \widehat{\Theta}\right) < \delta^{**}$. The proof of (q^{**}, m^{**}) on $\widehat{\Theta}$ satisfying ε-IM and IC is identical to the proof of Lemma 1. To see that (q^{**}, m^{**}) on $\widehat{\Theta}$ satisfies BB, observe that

$$\sum_{i \in I} m_i^{**}\left(\widehat{\theta}\right) = \sum_{i \in I} m_i(\theta) = 0, \forall \widehat{\theta} \in \widehat{\Theta},$$

where the first equality follows from (27) and the second equality follows from (22). Hence, $\widehat{\Theta} \in \mathcal{T}^{\varepsilon\text{-IM,BB,IC}}$.■

Proof of Lemma 5 Recall that $f(v(\theta))$ is continuous at every $\theta \in \Theta$ and $|\Theta| < \infty$. Hence, q^* and $(m_i^{**})_{i \in I}$ are continuous at every $\theta \in \Theta$. It follows that there exists $\delta^* \in \left(0, \frac{\vartheta}{4}\right)$ such that for any any $\widehat{\Theta} \in \mathcal{P}$ with $d_{\mathcal{T}}\left(\Theta, \widehat{\Theta}\right) < \delta^*$ and any $\left(\theta, \theta', \widehat{\theta}, \widehat{\theta}'\right) \in \Theta \times \Theta \times \widehat{\Theta} \times \widehat{\Theta}$, we have

$$\max\left\{d\left(\theta, \widehat{\theta}\right), d\left(\theta', \widehat{\theta}'\right)\right\} < \delta^* \implies \left|U_i\left(\theta_i' | \theta_i, q^*, m^{**}\right) - U_i\left(\widehat{\theta}_i' | \widehat{\theta}_i, q^*, m^{**}\right)\right| < \frac{\varepsilon}{4}. \quad (29)$$

We now prove

$$\left\{\widehat{\Theta} \in \mathcal{T} : d_{\mathcal{T}}\left(\Theta, \widehat{\Theta}\right) < \delta^*\right\} \subset \mathcal{T}^{\text{IM,BB},\varepsilon\text{-IC}}.$$

Fix any $\widehat{\Theta} \in \mathcal{T}$ such that $d_{\mathcal{T}}\left(\Theta, \widehat{\Theta}\right) < \delta^*$. The proof of (q^*, m^{**}) on $\widehat{\Theta}$ satisfying IM and ε-IC is identical to the proof of Lemma 2. To see that (q^*, m^{**}) on $\widehat{\Theta}$ satisfies BB, observe that

$$\sum_{i \in I} m_i^{**}\left(\widehat{\theta}\right) = \sum_{i \in I} m_i(\theta) = 0, \forall \widehat{\theta} \in \widehat{\Theta},$$

where the first equality follows from (27) and the second equality follows from (22). Hence, $\widehat{\Theta} \in \mathcal{T}^{\text{IM,BB},\varepsilon\text{-IC}}$.■

References

AKERLOF, G. A. (1970): “The Market for "Lemons": Quality Uncertainty and the Market Mechanism,” *The Quarterly Journal of Economics*, 84, 488–500.

ARROW, K. (1979): "The property rights doctrine and demand revelation under incomplete information," in *Economics and Human Welfare*, ed. by M. Boskin. Academic Press, New York.

BÖRGERS, T. (2013): "(No) Foundations of Dominant-Strategy Mechanisms: A Comment On Chung and Ely (2007)," Working paper.

CHE, Y.-K., AND J. KIM (2006): "Robustly Collusion-Proof Mechanisms," *Econometrica*, 74, 1063–1107.

CHEN, Y.-C., AND S. XIONG (2011): "The genericity of beliefs-determine-preferences models revisited," *Journal of Economic Theory*, 146, 751–761.

——— (2013a): "Genericity and Robustness of Full Surplus Extraction," *Econometrica*, 81, 825–847.

——— (2013b): "Supplement to 'Genericity and Robustness of Full Surplus Extraction'," *Econometrica Supplementary Material*, 81.

CRÉMER, J., AND R. MCLEAN (1988): "Full Extraction of the Surplus in Bayesian and Dominant Strategy Auctions," *Econometrica*, 53, 345–361.

D'ASPREMONT, C., J. CRÉMER, AND L.-A. GÉRARD-VARET (2003): "Correlation, independence, and Bayesian incentives," *Social Choice and Welfare*, 21(2), 281–310.

D'ASPREMONT, C., AND L.-A. GÉRARD-VARET (1979): "Incentives and Incomplete Information," *Journal of Public Economics*, 11, 25–45.

DEKEL, E., D. FUDENBERG, AND S. MORRIS (2006): "Topologies on Types," *Theoretical Economics*, 1, 275Ű309.

ELY, J. C., AND M. PESKI (2011): "Critical Types," *The Review of Economic Studies*, 78, 907–937.

HEIFETZ, A., AND Z. NEEMAN (2006): "On the Generic (Im)Possibility of Full Surplus Extraction in Mechanism Design," *Econometrica*, 74, 213–233.

HELLMAN, Z. (2014): "A Game with No Approximate Bayesian Equilibrium," *Journal of Economic Theory*, 153, 138–151.

HURWICZ, L. (1960): "Optimality and Informational Efficiency in Resource Allocation Processes," in *Mathematical Methods in the Social Sciences*, ed. by K. Arrow, S. Karlin, and P. Suppes. Stanford University Press.

HURWICZ, L. (1972): "On Informationally Decentralized Systems," in *Decision and Organization*, ed. by C. McGuire, and R. Radner. North Holland, Amsterdam.

LAFFONT, J.-J., AND D. MARTIMORT (2000): "Mechanism Design With Collusion and Correlation," *Econometrica*, 68, 309–342.

MCAFEE, R. P., AND P. J. RENY (1992): "Correlated Information and Mecanism Design," *Econometrica*, 60, 395–421.

MERTENS, J. F., AND S. ZAMIR (1985): "Formulation of Bayesian analysis for games with incomplete information," *International Journal of Game Theory*, 14, 1–29.

MYERSON, R., AND M. SATTERTHWAITE (1983): "Efficient Mechanisms for Bilateral Trading," *Journal of Economic Theory*, 29, 265–281.

MYERSON, R. B. (1981): "Optimal Auction Design," *Mathematics of Operations Research*, 6, 58–73.

PETERS, M. (2001): "Surplus Extraction and Competition," *Review of Economic Studies*, 68, 613–631.

ROBERT, J. (1991): "Continuity in Auction Design," *Journal of Economic Theory*, 55, 169–179.

ROTHSCHILD, M., AND J. E. STIGLITZ (1976): "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information," *The Quarterly Journal of Economics*, 90, 629–649.

SIMON, R. (2003): "Games of incomplete information, ergodic theory, and the measurability of equilibria," *Israel journal of mathematics*, 138, 73–92.

SPENCE, M. (1973): "Job Market Signaling," *The Quarterly Journal of Economics*, 87, 355–374.