Genericity and Robustness of Full Surplus Extraction*

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Abstract

We study whether priors that admit full surplus extraction (FSE) are generic, which becomes a gauge to evaluate the validity of the current mechanism design paradigm. We consider the space of priors on the universal type space, and thereby relax the assumption of a fixed finite number of types in Crémer and McLean (1988) along the direction of *Wilson's Doctrine*. We show that FSE priors are topologically generic, contrary to the result of Heifetz and Neeman (2006) that FSE is generically impossible, both geometrically and measure-theoretically. Instead of using the BDP approach or convex combination of priors adopted in Heifetz and Neeman (2006), we prove our genericity results by showing a robustness property of *Crémer-McLean mechanisms*.

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1 Introduction

In economic models, agents with private information about their independent values retain some informational rent (Myerson (1981)). However, the source of the informational rent is not privacy; rather, it is the independence of information among agents. In a seminal paper, Crémer and McLean (1988) (hereafter, CM) show that in a classical mechanism design model (hereafter, the classical model), a mechanism designer can *fully* extract agents' rent even if their values are only slightly correlated. Since *nearly all* models have correlated information, full surplus extraction (FSE) should be a generic phenomenon. However, FSE is rarely observed in reality. This raises questions about the validity of the classical model to approximate reality. As McAfee and Reny note:

> The results (full rent extraction) cast doubt on the value of the current mechanism design paradigm as a model of institution design (McAfee and Reny, 1992, p.400).[1]

Many explanations have been proposed to resolve the puzzle. We know now that CM's genericity result does not hold if any of the following essential assumptions of the classical model is violated — risk neutrality, unlimited liability, absence of collusion among agents, and lack of competition among sellers (see Robert (1991), Laffont and Martimort (2000), and Peters (2001)).

However, the classical model that incorporates *all* these assumptions is still the most commonly used model. Although the puzzle on the genericity of FSE is resolved, the doubt on the validity of the classical model remains. By modifying the essential assumptions of the classical model, the aforementioned explanations only demonstrate that the *modified* models are immune to the critique raised by the genericity of FSE (hereafter the FSE critique), but do not explain why the classical model itself generates predictions that contradict our observations. The issue can only be satisfactorily resolved if we can attribute the genericity of FSE to assumptions which are *inessential* to the classical model and yet is critical to the genericity of FSE. Failing to find such inessential assumptions would invite re-examination of the classical model and all the theories based on it.

[1] McAfee and Reny (1992) fully charaterize FSE in a fixed type space with a continuum of types and continuous density functions.

In an important paper, Heifetz and Neeman (2006) (hereafter, HN) identify such an assumption. In particular, HN point out that CM's result hinges on an implicit common-knowledge assumption that each agent has a fixed finite number of types. This assumption, in spite of being necessary for CM's genericity result, is not essential to the classical model. Indeed, there is no *a priori* finite bound for the number of types when we model a situation involving asymmetric information. Following *Wilson's Doctrine* that "only by repeated weakening of common knowledge assumptions will the theory approximate reality," HN relax the assumption and prove that FSE priors are "small" in a geometric sense (i.e., they are contained in a proper face), and a measure-theoretical sense (i.e., they are contained in a finitely shy set as defined in Anderson and Zame (2001)).[2]

In this paper, we also relax CM's common-knowledge assumption, and yet we prove that FSE is topologically generic.[3] We thus provide a sense in which FSE is still generic under the classical model even if we relax the inessential common-knowledge assumption in CM. More importantly, our results imply that the classical model remains subject to the FSE critique.

Specifically, we study (common) priors on the universal type space (see Mertens and Zamir (1985) and Heifetz and Neeman (2006)). The universal type space is known to embed all (Harsanyi) type spaces, therefore, our approach not only relaxes CM's common-knowledge assumption, but also entails no loss of generality (see Section 4.2 for a formal discussion). Following Mertens, Sorin, and Zamir (1994), we endow the space of priors with the standard weak* topology.[4] We say a set is (topologically) generic if it contains a residual set, i.e., it contains a countable intersection of open and dense sets; a set is nongeneric if its complement is generic, i.e., it is contained in a meager set.[5] We report

[2] Finite shyness is a notion which extends the notion of shyness originally proposed in Hunt, Sauer, and Yorke (1992). See Anderson and Zame (2001) for details.

[3] We compare our paper with HN in details in Section 4.1.

[4] Agents choose their best strategy according to their expected utility, and the mechanism designer chooses the optimal mechanism according to her expected revenue. The weak* topology is the coarsest topology which makes these "expected values" continuous in beliefs. Barelli (2009) and Chen and Xiong (2011) also adopt the weak* topology on priors. Moreover, the definition of the measure-theoretic notion of genericity adopted in HN also requires a topology on priors and HN take the weak* topology as an important candidate. See Section 4.3 for more discussion on topology.

[5] Being residual and meager are standard notions of topological genericity (being typical) and nongenericity (being negligible), respectively. This notion of genericity is also adopted in Barelli (2009), Chen and

two genericity results. We first consider the space of all finite priors: A prior is finite if its support is a finite set, and infinite otherwise. Our first result is that there is a generic set of finite priors such that each prior in the set admits a mechanism that achieves FSE, and moreover, it is dominant strategy incentive compatible and individually rational. We then consider the space of all priors, which can be either finite or infinite. To address the technical difficulties associated with infinite priors, we follow McAfee and Reny (1992) and consider *almost FSE* (AFSE). A prior is AFSE if for any $\varepsilon > 0$, we can construct a mechanism that extracts $1 - \varepsilon$ of the total surplus, and moreover, it is Bayesian incentive compatible and individually rational. Our second result is that AFSE is generic among all priors.

Unlike HN, we prove our genericity results by directly studying the mechanism design problems on general priors.[6] We consider a class of mechanisms called *Crémer-McLean mechanisms* (CM mechanisms). A (continuous) CM mechanism is a second-price auction together with side payments that are determined (continuously) by other agents' reported types. The gist of our proof is a robustness property for continuous CM mechanisms: If a continuous CM mechanism extracts more than $1 - \varepsilon$ of the total surplus on a prior μ, it would continue to do so for any prior in a small weak*-neighborhood of μ. With this robustness property, the intuition of our genericity results become transparent. Let B_n denote the set of priors for which more than $1 - 1/n$ of the total surplus can be extracted via some continuous CM mechanism. B_n is thus open by the robustness property, and dense by the previous results in Crémer and McLean (1988) and Mertens, Sorin, and Zamir (1994). Since the set of (A)FSE priors contains the residual set $\cap_n B_n$, it follows that (A)FSE is generic.

Finally, we note that the opposite conclusions in HN and this paper are solely due to the different perspectives we take, i.e., HN take the geometric/measure-theoretical approach and we take the topological approach. Since we study an infinite-dimensional space of priors for which there is no consensus on the notion of genericity, the answer to which approach is better seems subjective.[7] Nevertheless, in Section 4.1.1, we provide

Xiong (2011), Dekel, Fudenberg, and Morris (2006), and Ely and Pęski (2011).

[6]HN's genericity result relies upon a property of a prior called *BDP* that is due to Neeman (2004). See Section 4.1.2 for details.

[7]Anderson and Zame (2001) point out some weakness of the residual (resp. meager) set as the notion of genericity (resp. nongenericity). Stinchcombe (2000) discusses some caveats of the prevlance (resp. shy) set as the notion of genericity (resp. nongenericity).

an important situation to which the topological approach is applicable while the other approach is not.

The rest of this paper is organized as follows. Section 2 contains notations and definitions. Section 3 presents our main results. Section 4 discusses related issues. Section 5 concludes.

2 Preliminaries

Throughout this paper, for any compact metric space X with the metric d_X, we endow X with the Borel σ-algebra. Let $\Delta(X)$ denote the topological space of all probability measures on the Borel σ-algebra with the weak* topology. The weak* topology is metrizable under the Prohorov distance defined as

$$\rho(\mu, \mu') = \inf\{\varepsilon > 0 : \mu(E) \leq \mu'(E^{\varepsilon}) + \varepsilon \text{ for every Borel set } E \subset X\}, \forall \mu, \mu' \in \Delta(X).$$

where $E^{\varepsilon} \equiv \{x' : \inf_{x \in E} d_X(x', x) < \varepsilon\}$ (see (Dudley, 2002, 11.3.3. Theorem)). All product spaces are endowed with the product topology and subspaces with the relative topology. The support of a probability measure $\mu \in \Delta(X)$, denoted by Suppμ, is the intersection of all closed sets with measure one under μ. For any finite set $F \subset X$, let $|F|$ denote the cardinality of F.

2.1 Priors and belief spaces

One object is for sale. Let I be a finite set of bidders. For simplicity, we assume that for every $i \in I$, $V_i = [0, 1]$ is the set of bidder i's possible values of the object endowed with the Euclidean topology. Let Θ_i^* be the compact metric private-value universal type space on $[0, 1]^{|I|}$, which contains all possible bidder i's types (see Mertens and Zamir (1985) and (Heifetz and Neeman, 2006, pp.228-229)). Let $v_i : \Theta_i^* \to V_i$ and $b_i : \Theta_i^* \to \Delta(\Theta_{-i}^*)$ be the continuous functions through which each $\theta_i \in \Theta_i^*$ identifies a value $v_i(\theta_i)$ and a belief $b_i(\theta_i)$ of bidder i. Let $\Theta^* = \Pi_{i \in I} \Theta_i^*$ be the space of all bidders' type profiles and $\Theta_{-i}^* = \Pi_{j \neq i} \Theta_j^*$ be the space of bidder i's opponents' type profiles. Let d_i denote the metric on Θ_i^*, d_{-i} the metric on Θ_{-i}^* , and d the metric on Θ^*, where $d_{-i}(\theta_{-i}, \theta'_{-i}) = \max_{j \neq i} d_j\left(\theta_j, \theta'_j\right)$

and $d(\theta,\theta') = \max_{j\in I} d_j\left(\theta_j,\theta'_j\right)$. For each $\theta \in \Theta^*$, we denote by θ_i the type of bidder i under θ and we often save the notation to write $v_i(\theta)$ and $b_i(\theta)$ instead of $v_i(\theta_i)$ and $b_i(\theta_i)$.

A *belief subspace* Θ is a nonempty and closed subset of Θ^* such that for any $\theta \in \Theta$, $\{\theta_i\} \times \mathrm{Supp} b_i(\theta)$ is a subset of Θ. For any $\mu \in \Delta\left(\Theta_i^* \times \Theta_{-i}^*\right)$, denote by μ_i the marginal distribution of μ on Θ_i^*. A probability measure $\mu \in \Delta(\Theta^*)$ is said to be a *prior* if for any bounded measurable function $\varphi : \Theta^* \to \mathbb{R}$,

$$\int_{\Theta_i^*}\left(\int_{\Theta_{-i}^*} \varphi\left(\theta_i,\theta'_{-i}\right) b_i(\theta_i)\left[d\theta'_{-i}\right]\right)\mu_i[d\theta_i] = \int_{\Theta^*}\varphi(\theta)\,\mu[d\theta], \forall i. \tag{1}$$

Let $\mathcal{P} \subset \Delta(\Theta^*)$ be the set of all priors endowed with the weak* topology metrized by the Prohorov metric $d_{\mathcal{P}}$. For any prior μ, the support of μ is a belief subspace and it is denoted by Θ^μ.[8] A prior μ is said to be a *finite prior* if Θ^μ is a finite set. Let $\mathcal{P}^f$ denote the space of all finite priors. For any set $E \subset \Theta^*$, let E_i and E_{-i} denote the projections of E into Θ_i^* and Θ_{-i}^* respectively. The following result will be used later.

Lemma 1 ((Mertens, Sorin, and Zamir, 1994, Theorem 3.1)) *$\mathcal{P}^f$ is dense in $\mathcal{P}$.*

2.2 Mechanisms and surplus extraction

A mechanism designer tries to sell the object to the agents (i.e., bidders) in I. By revelation principle, it is without loss of generality to restrict our attention to direct mechanisms. A (direct) mechanism on a belief subspace Θ is a list of measurable functions $(q,m) \equiv \left(q_i : \Theta \to [0,1], \; m_i : \Theta \to \mathbb{R}\right)_{i\in I}$ satisfying $\sum_{i\in I} q_i(\theta) \leq 1$ for any $\theta \in \Theta$. For each profile of reports $\theta \in \Theta$, $q_i(\theta)$ specifies the probability that bidder i wins the object and $m_i(\theta)$ specifies how much bidder i pays.

Fix any mechanism (q,m) defined on a belief subspace Θ. Let $u_i\left(\theta'_i,\theta_{-i}|\theta_i,q,m\right)$ denote the *ex post* payoff of type θ_i, when he reports θ'_i and the other bidders report $\theta_{-i} \in \Theta_{-i}$, i.e.,

[8] See (Mertens, Sorin, and Zamir, 1994, p.147, item 2). In the literature, a belief subspace which is the support for some common prior is called a *consistent belief subspace*. Since we restrict our attention to consistent belief subspaces, we will omit "consistent" for simplicity.

$$u_i\left(\theta_i', \theta_{-i}|\theta_i, q, m\right) \equiv v_i\left(\theta_i\right) q_i\left(\theta_i', \theta_{-i}\right) - m_i\left(\theta_i', \theta_{-i}\right); \tag{2}$$

Furthermore, let $U_i\left(\theta_i'|\theta_i, q, m\right)$ denote the interim expected payoff of type θ_i when he reports θ_i' and the other bidders truthfully reveal their types, i.e.,

$$U_i\left(\theta_i'|\theta_i, q, m\right) = \int_{\Theta_{-i}} u_i\left(\theta_i', \theta_{-i}|\theta_i, q, m\right) b_i\left(\theta_i\right)\left[d\theta_{-i}\right].$$

To simplify our notation, we will write $U_i\left(\theta_i|q, m\right)$ for $U_i\left(\theta_i|\theta_i, q, m\right)$, which is the interim expected payoff of type θ_i when all bidders truthfully reveal their types.

Definition 1 *A mechanism (q, m) defined on a belief subspace Θ is individually rational (IR) if*

$$U_i\left(\theta_i|q, m\right) \geq 0 \text{ for every } (i, \theta) \in I \times \Theta.$$

Definition 2 *A mechanism (q, m) defined on a belief subspace Θ is dominant strategy incentive compatible (DSIC) if*

$$u_i\left(\theta_i, \theta_{-i}|\theta_i, q, m\right) \geq u_i\left(\theta_i', \theta_{-i}|\theta_i, q, m\right) \text{ for every } \left(i, \theta, \theta'\right) \in I \times \Theta \times \Theta. \tag{3}$$

That is, (q, m) on Θ satisfies DSIC iff truthful reporting is a weakly dominant strategy for every bidder.[9]

Definition 3 *A mechanism (q, m) defined on a belief subspace Θ is Bayesian incentive compatible (BIC) if*

$$U_i\left(\theta_i|q, m\right) \geq U_i\left(\theta_i'|\theta_i, q, m\right) \text{ for every } \left(i, \theta, \theta'\right) \in I \times \Theta \times \Theta.$$

That is, (q, m) on Θ satisfies BIC iff truthful reporting constitutes a Bayesian Nash equilibrium on Θ. Clearly, if (q, m) is DSIC, then it is also BIC.

Definition 4 *For any $\varepsilon \geq 0$, a mechanism (q, m) defined on a belief subspace Θ satisfies ε-*surplus-extraction (ε-SE) *if*

$$\left|\int_{\Theta}\left[\sum_{i \in I} m_i(\theta) - \max_{i \in I} v_i(\theta)\right] \mu[d\theta]\right| \leq \varepsilon.$$

[9](3) is usually called *ex post* Incentive Compatibility (EPIC). In the private-value environments considered in this paper, DSIC and EPIC are equivalent, see Bergemann and Morris (2005).

Note that $\int \max_{i\in I} v_i(\theta)\,\mu[d\theta]$ is the maximal value bidders can achieve by getting the object and $\int \sum_{i\in I} m_i(\theta)\,\mu(d\theta)$ is the payment collected from the bidders. Hence, if (q,m) satisfies 0-SE, it fully extracts bidders' rents.

We say (q,m) is efficient at θ if the object is always allocated to the bidder(s) with the highest value, i.e.,

$$\sum_{\left\{i\in I: v_i(\theta)\geq v_j(\theta),\forall j\in I\right\}} q_i(\theta) = 1. \tag{4}$$

By (2) and (4), if (q,m) is efficient at θ, we have

$$\sum_{i\in I} u_i(\theta|\theta_i,q,m) \equiv \max_{i\in I} v_i(\theta) - \sum_{i\in I} m_i(\theta). \tag{5}$$

Furthermore, we say (q,m) is efficient on E, if (q,m) is efficient at every $\theta\in E$. Then,

$$\begin{aligned} & \left|\int_E \left[\sum_{i\in I} m_i(\theta) - \max_{i\in I} v_i(\theta)\right]\mu[d\theta]\right| \\ = & \left|\int_E \sum_{i\in I} u_i(\theta|\theta_i,q,m)\,\mu[d\theta]\right| \\ = & \left|\sum_{i\in I}\int_{E_i}\left[\int_{E_{-i}} u_i(\theta|\theta_i,q,m)\,b_i(\theta_i)[d\theta_{-i}]\right]\mu_i[d\theta_i]\right| \end{aligned} \tag{6}$$

where the first equality follows from (5); the second follows from (1). In particular, if (q,m) is efficient on Θ^μ and take $E=\Theta^\mu$, we get the following equality by the definition of $U_i(\theta_i|q,m)$.

$$\left|\int_{\Theta^\mu}\left[\sum_{i\in I} m_i(\theta) - \max_{i\in I} v_i(\theta)\right]\mu[d\theta]\right| = \left|\sum_{i\in I}\int_{\Theta_i^\mu}\left[U_i(\theta_i|q,m)\right]\mu_i[d\theta_i]\right|$$

That is, for an IR mechanism (q,m) which is efficient on Θ^μ, (q,m) is 0-SE if and only if for every i, $U_i(\theta_i|q,m)=0$ for μ_i-almost all θ_i.

Let (q^*,m^*) denote the second-price (sealed-bid) auction defined on Θ^* with an arbitrary tie-breaking rule. That is, (q^*,m^*) satisfies

1. $q_i^*(\theta) > 0$ only if $v_i(\theta)\geq v_j(\theta)$ for any $j\in I$;
2. $m_i^*(\theta) = q_i^*(\theta)\times\max_{j\neq i} v_j(\theta)$.

It is well known that (q^*, m^*) is efficient, IR, and DSIC (and hence also BIC).

We now define two notions of full surplus extraction.

Definition 5 *A prior μ is a* full-surplus-extraction (FSE) *prior if there exists a mechanism (q, m) on Θ^μ which is IR, DSIC and 0-SE.*

Definition 6 *A prior μ is an* almost-full-surplus-extraction (AFSE) *prior if for any $\varepsilon > 0$, there exists a mechanism (q, m) on Θ^μ which is IR, BIC, and ε-SE.*

We use $\mathcal{F}$ and $\mathcal{F}^f$ to denote the sets of FSE priors in $\mathcal{P}$ and $\mathcal{P}^f$ respectively. Thus, $\mathcal{F}^f = \mathcal{F} \cap \mathcal{P}^f$. We use $\mathcal{A}$ to denote the set of AFSE priors in $\mathcal{P}$. We require DSIC for FSE and BIC for AFSE. We make the difference so as to accommodate some technical difficulties associated with infinite priors.[10]

2.3 Genericity

In a topological space X, a set is called a nowhere dense set if its closure contains no interior point. A set $E \subset X$ is called a meager set if $E = \cup_{n=1}^{\infty} E_n$ such that every E_n is a nowhere dense set. A set $F \subset X$ is called a residual set if $X \backslash F$ is a meager set.

Definition 7 *In a topological space X, we say $F \subset X$ is generic if F contains a residual set. We say $E \subset X$ is nongeneric if $X \backslash E$ is generic (i.e., E is contained in a meager set).*

3 Main results

In this section, we present our genericity results. We first study the space of finite priors $\mathcal{P}^f$ and show that full surplus extraction is generic in $\mathcal{P}^f$ (Theorem 1). We then study the space of all priors $\mathcal{P}$ and show that almost full surplus extraction is generic in $\mathcal{P}$

[10] McAfee and Reny (1992) also adopt $\mathcal{A}$ rather than $\mathcal{F}$ for an infinite prior with a continuous density function.

(Theorem 2). Before presenting the genericity results, we define a class of mechanisms called Crémer-McLean mechanisms (CM mechanisms) and provide several intermediate lemmas which are useful for our proofs.

3.1 Crémer-McLean mechanism and its robustness property

Recall that (q^*, m^*) denotes the second-price auction.

Definition 8 *A mechanism (q, m) on a belief subspace Θ is called a Crémer-McLean (CM) mechanism if there exists $w_i : \Theta_{-i} \to \mathbb{R}$ such that $q_i(\theta) = q_i^*(\theta)$ and $m_i(\theta) = m_i^*(\theta) + w_i(\theta_{-i})$ for every $(i, \theta) \in I \times \Theta$. Furthermore, (q, m) is a continuous CM mechanism on Θ if w_i is continuous for every $i \in I$.*

The function w_i is often called a side-payment scheme for bidder i. A distinct feature of w_i is that it only depends on bidder $-i$'s report, but not on i's report. Due to this feature, truthful reporting is still a weakly dominant strategy for every type under a CM mechanism (as in a second-price auction):

Lemma 2 *Every CM mechanism satisfies DSIC.*

Define

$$\mathcal{P}_n^f \equiv \left\{ \mu \in \mathcal{P}^f \colon \left| \Theta_i^\mu \right| = n, \forall i \in I \right\}.$$

That is, $\mathcal{P}_n^f$ is the set of priors whose supports are belief subspaces containing exactly n types for each bidder. Pick any $\mu \in \mathcal{P}_n^f$. Define the interim belief matrix for bidder i as

$$\mathbf{B}_i^\mu \equiv \left[b_i(\theta_i)[\theta_{-i}] \right]_{\theta_i \in \Theta_i^\mu,\, \theta_{-i} \in \Theta_{-i}^\mu}$$

where each row corresponds to a type θ_i of bidder i and her belief about the distribution of her opponents' types, i.e., $\left[b_i(\theta_i)[\theta_{-i}] \right]_{\theta_{-i} \in \Theta_{-i}^\mu}$. We say $\mathbf{B}_i^\mu$ has *full rank* if its column space has rank n, and μ has full rank if $\mathbf{B}_i^\mu$ has full rank for every $i \in I$.

The FSE result in Crémer and McLean (1988) implies the following lemma.

Lemma 3 *$\mu \in \mathcal{F}$ if $\mu \in \mathcal{P}_n^f$ has full rank.*

Clearly, the set of priors (in $\mathcal{P}_n^f$) that have full rank is open and dense in $\mathcal{P}_n^f$. This then yields the genericity result in Crémer and McLean (1988) which says that $\mathcal{P}_n^f \cap \mathcal{F}^f$ is generic in $\mathcal{P}_n^f$. However, as argued in Heifetz and Neeman (2006), there is no *a priori* finite bound for the number of types when we model a situation involving asymmetric information. Hence, we should relax this common-knowledge assumption on the size of priors' supports, which leads to the question: is $\mathcal{F}^f$ generic in $\mathcal{P}^f$? We provide a positive answer to this question in Section 3.2.

Define

$$\mathcal{F}^{f,cm} \equiv \left\{ \mu \in \mathcal{P}^f : \exists \text{ a CM mechanism on } \Theta^\mu \text{ which is IR, DSIC, and 0-SE} \right\}. \tag{7}$$

Lemma 4 *$\mathcal{F}^{f,cm}$ is dense in both $\mathcal{P}^f$ and $\mathcal{P}$.*

Lemma 4 is an immediate consequence of the following facts: i) $\mathcal{P}^f$ is dense in $\mathcal{P}$ (Lemma 1); ii) $\cup_{n=1}^{\infty} \mathcal{P}_n^f$ is dense in $\mathcal{P}^f$;[11] iii) priors having full rank are dense in $\mathcal{P}_n^f$; iv) for any prior $\mu \left(\in \mathcal{P}_n^f \right)$ that has full rank, there is some CM mechanism on Θ^μ which is IR, DSIC, and 0-SE (Lemma 3).

However, as discussed above, denseness alone is not a good notion of genericity. It can be shown that $\mathcal{P}^f \backslash \mathcal{F}^{f,cm}$ is also dense in $\mathcal{P}^f$ and hence also in $\mathcal{P}$ (by Lemma 1).[12]

Lemma 5 *For any $\mu \in \mathcal{P}$ and any continuous CM mechanism (q, m) on Θ^μ, there exists a continuous CM mechanism (q', m') on Θ^* such that $q'(\theta) = q(\theta)$ and $m'(\theta) = m(\theta)$ for all $\theta \in \Theta^\mu$.*

[11] To see the idea, consider the case with two bidders. In the following examples, $\mu \in \mathcal{P}^f$ and $\mu \notin \mathcal{P}_n^f$ for any n, while $\mu_m \in \mathcal{P}_2^f$ for all m and $\mu_m \to \mu$ as $m \to \infty$.

μ:

μ	$v_2 = 0$
$v_1 = 0$	$\frac{3}{4}$
$v_1 = 1$	$\frac{1}{4}$

and μ_m:

μ	$v_2 = 0$	$v_2 = 1$
$v_1 = 0$	$\left(1 - \frac{1}{m}\right) \frac{3}{4}$	$\frac{1}{2m}$
$v_1 = 1$	$\left(1 - \frac{1}{m}\right) \frac{1}{4}$	$\frac{1}{2m}$

[12] Bergemann and Morris (2001) show a closely-related result that both what they call Neeman-types and non-Neeman types are dense in the universal type space endowed with product topology.

Lemma 5 is an immediate consequence of the Tietze Extension Theorem (see (Aliprantis and Border, 2006, 2.47)). In particular, if μ is a finite prior, any CM mechanism (q, m) on Θ^{μ} is trivially continuous and hence can be extended as a continuous CM mechanism on Θ^* by Lemma 5.

Lemma 6 *Given any continuous CM mechanism (q, m) on Θ^*, $U_i(\cdot|q, m) : \Theta_i^* \to \mathbb{R}$ is uniformly continuos on Θ_i^*. Moreover, $\max_{\theta \in \Theta^*} |m_i(\theta)| < \infty$ and $\max_{\theta \in \Theta^*} |u_i(\theta|\theta_i, q, m)| < \infty$.*

Lemma 6 is due to continuity of payoffs and compactness of Θ^*. The proof can be found in A.1.

Proposition 1 below establishes an important robustness property of a continuous CM mechanism: If a continuous CM mechanism extracts more than $1 - \varepsilon$ of the total surplus on a prior μ, it would continue to do so for any prior in a small weak*-neighborhood of μ. This result implies that surplus extraction using a continuous CM mechanism is robust to slight misspecifications of the prior.

Proposition 1 *For any $r' > 0$, $1 > r > 0$, and any continuous CM mechanism (q, m) on Θ^*, let*

$$\mathcal{H} \equiv \left\{ \mu \in \mathcal{P} \colon \begin{array}{l} \exists \text{ compact set } E \subset \Theta^* \text{ s.t.} \\ i)\ \mu(E) > 1 - r; \\ ii)\ U_i(\theta_i|q, m) \in (0, r'),\ \forall (i, \theta) \in I \times E \end{array} \right\}.$$

Then, $\mathcal{H}$ is open.

The relationship between $\mathcal{H}$ and almost full surplus extraction needs some explanation. For sufficiently small r', $U_i(\theta_i|q, m) \in (0, r')$ means that little rent is surrendered to type θ_i. For sufficiently small r, $\mu(E) > 1 - r$ means that almost full surplus extraction occurs with a probability sufficiently close to 1. Though the proof of Proposition 1 is involved, Lemma 6 provides a straightforward intuition: with a continuous CM mechanism, the extracted rent is continuous with respect to priors. The proof can be found in A.2.

3.2 Genericity of full surplus extraction in the space of finite priors

Theorem 1 *$\mathcal{F}^f$ is generic in $\mathcal{P}^f$. That is, full surplus extraction is generically possible in the space of finite priors.*

Define

$$\mathcal{D}_n = \left\{ \mu \in \mathcal{P}^f : \begin{array}{l} \exists K \subset \Theta^\mu \text{ and a CM mechanism } (q, m) \text{ on } \Theta^\mu \text{ s.t.} \\ \text{i) } \mu(K) > 1 - \frac{1}{n}; \\ \text{ii) } U_i(\theta_i | q, m) \in \left(0, \frac{1}{n}\right), \forall (i, \theta) \in I \times K. \end{array} \right\}. \tag{8}$$

Theorem 1 is a direct consequence of the following three results which will be proved in Sections 3.2.1, 3.2.2 and 3.2.3.

(3.2.1) $\mathcal{D}_n$ is dense in $\mathcal{P}^f$;

(3.2.2) $\mathcal{D}_n$ is open in $\mathcal{P}^f$;

(3.2.3) $\cap_{n=1}^{\infty} \mathcal{D}_n \subset \mathcal{F}^f$.

An immediate corollary of Theorem 1 is that $\mathcal{F}$ cannot be nongeneric in $\mathcal{P}$.

Corollary 1 *$\mathcal{F}$ is not non-generic in $\mathcal{P}$. That is, full surplus extraction is not generically impossible in the space of all priors.*

Corollary 1 is implied by the following fact: If Y is a dense subset of X, then for any $U (\subset X)$ which is open and dense in X, $Y \cap U$ is open and dense in Y. As a result, all residual sets and meager sets in X can be carried over to a dense subspace of X. Hence, if $\mathcal{F}$ were a nongeneric set in $\mathcal{P}$, then $\mathcal{F}^f$ would also be a nongeneric set in $\mathcal{P}^f$, which contradicts Theorem 1. A detailed proof of Corollary 1 is provided in A.3.[13]

[13]Whether $\mathcal{F}$ is generic in $\mathcal{P}$ remains an open question to us.

3.2.1 $\mathcal{D}_n$ is dense in $\mathcal{P}^f$

Recall the set $\mathcal{F}^{f,cm}$ defined in (7). We show $\mathcal{F}^{f,cm} \subset \mathcal{D}_n$ and hence $\mathcal{D}_n$ is dense by Lemma 4. To see $\mathcal{F}^{f,cm} \subset \mathcal{D}_n$, pick any finite prior $\mu \in \mathcal{F}^{f,cm}$. By the definition of $\mathcal{F}^{f,cm}$ in (7), there exists a CM mechanism (q, m) on Θ^μ such that $U_i(\theta_i | q, m) = 0$ for every $(i, \theta) \in I \times \Theta^\mu$.

Define a new CM mechanism (q', m') on Θ^μ such that $q'_i(\theta) = q_i(\theta)$ and $m'_i(\theta) = m_i(\theta) - \frac{1}{2n}$ for any $(i, \theta) \in I \times \Theta^\mu$. That is, the only difference between (q, m) and (q', m') is that the side payment for every bidder in (q', m') is always $\frac{1}{2n}$ less than the side payment for every bidder in (q, m). As a result, $U_i(\theta_i | q', m') = \frac{1}{2n}$ for every $(i, \theta) \in I \times \Theta^\mu$.

Under the CM mechanism (q', m'), consider $K = \Theta^\mu$. We thus have both i) and ii) in the definition of $\mathcal{D}_n$ in (8). Hence, $\mu \in \mathcal{D}_n$. Therefore, $\mathcal{F}^{f,cm} \subset \mathcal{D}_n$.$\blacksquare$

3.2.2 $\mathcal{D}_n$ is open in $\mathcal{P}^f$

Pick any $\mu \in \mathcal{D}_n$. Then, there exits a (finite and hence) compact set $K \subset \Theta^\mu$, and a CM mechanism (q, m) on Θ^μ such that $\mu(K) > 1 - \frac{1}{n}$ and $U_i(\theta_i | q, m) \in \left(0, \frac{1}{n}\right)$, $\forall (i, \theta) \in I \times K$. Since Θ^μ is finite, the CM mechanism (q, m) on Θ^μ is trivially a continuous CM mechanism. Hence, by Lemma 5, we can extend the CM mechanism (q, m) on Θ^μ to a continuous CM mechanism (q', m') on Θ^*.

Then, with $r' = r = \frac{1}{n}$ and the continuous mechanism CM (q', m') on Θ^*, the set $\mathcal{H}_\mu$ defined below is open in $\mathcal{P}$ by Proposition 1.

$$\mathcal{H}_\mu \equiv \left\{ \mu' \in \mathcal{P}\text{:} \begin{array}{l} \exists \text{ compact set } E \subset \Theta^* \text{ s.t.} \\ \text{i) } \mu'(E) > 1 - \frac{1}{n}; \\ \text{ii) } U_i(\theta_i | q', m') \in \left(0, \frac{1}{n}\right), \forall (i, \theta) \in I \times E \end{array} \right\}.$$

Consequently, $\mu \in \mathcal{H}_\mu \cap \mathcal{P}^f \subset \mathcal{D}_n$. Since $\mathcal{H}_\mu \cap \mathcal{P}^f$ is open in $\mathcal{P}^f$, it follows that $\mathcal{D}_n$ is open in $\mathcal{P}^f$.$\blacksquare$

3.2.3 $\cap_{n=1}^{\infty}\mathcal{D}_n \subset \mathcal{F}^f$

Fix any $\mu \in \cap_{n=1}^{\infty}\mathcal{D}_n$ and we will show $\mu \in \mathcal{F}^f$. Since μ is fixed in the proof, for simplicity we write Θ instead of Θ^μ for the support of μ and $\mathbf{B}_i$ instead of $\mathbf{B}_i^\mu$ for the interim belief matrix for bidder i. Recall $\mathbf{B}_i \equiv [b_i(\theta_i)[\theta_{-i}]]_{\theta_i\in\Theta_i,\,\theta_{-i}\in\Theta_{-i}}$. For any CM mechanism (q,m) with $q_i(\theta) = q_i^*(\theta)$ and $m_i(\theta) = m_i^*(\theta) + w_i(\theta_{-i})$, define

$$\mathbf{U}_i^{(q,m)} \equiv [U_i(\theta_i|q,m)]_{\theta_i\in\Theta_i};$$
$$\mathbf{W}_i^{(q,m)} \equiv [w_i(\theta_{-i})]_{\theta_{-i}\in\Theta_{-i}},$$

where $\mathbf{U}_i^{(q,m)}$ is the column vector of the interim expected payoffs for bidder i's types in the support of μ under (q,m) and $\mathbf{W}_i^{(q,m)}$ is the column vector of side-payments $w_i(\theta_{-i})$ indexed by $\theta_{-i} \in \Theta_{-i}$. Hence, we have

$$\mathbf{U}_i^{(q,m)} = \mathbf{U}_i^{(q^*,m^*)} - \mathbf{B}_i\mathbf{W}_i^{(q,m)}.$$

Since μ is a finite prior, Θ is a finite set. Thus, $\alpha \equiv \min_{\theta\in\Theta}\mu[\theta] > 0$. For any n with $\frac{1}{n} < \alpha$, by $\mu \in \mathcal{D}_n$ and the definition of $\mathcal{D}_n$, there exist $K \subset \Theta^\mu$ and some CM mechanism (q,m_n) such that

i) $\mu(K) > 1 - \frac{1}{n}$;

ii) $U_i(\theta_i|m_n,q) \in \left(0,\frac{1}{n}\right), \forall (i,\theta) \in I \times K.$

Since $\alpha = \min_{\theta\in\Theta}\mu[\theta]$ and $\frac{1}{n} < \alpha$, we conclude that $K = \Theta^\mu$ from condition i). Then, condition ii) implies

$$\left\|\mathbf{U}_i^{(q,m_n)}\right\|_\infty = \left\|\mathbf{U}_i^{(q^*,m^*)} - \mathbf{B}_i\mathbf{W}_i^{(q,m_n)}\right\|_\infty \in [0,1/n] \text{ for any } i \in I \text{ and any } n > \frac{1}{\alpha}, \quad (9)$$

where $\|\cdot\|_\infty$ is the supmetric in Euclidean space, i.e., $\|(a_1,...,a_k)\|_\infty = \max\{|a_1|,...,|a_k|\}$.

The vector $\mathbf{B}_i\mathbf{W}_i^{(q,m_n)}$ is contained in the column space of $\mathbf{B}_i$. Since (9) is true for any $n > \frac{1}{\alpha}$, $\mathbf{U}_i^{(q^*,m^*)}$ is in the closure of the column space of $\mathbf{B}_i$. Since the column space of a finite matrix is a finite-dimensional linear space which is closed (see (Aliprantis and Border, 2006, 5.22 Corollary)), we conclude that $\mathbf{U}_i^{(q^*,m^*)}$ is in the column space of $\mathbf{B}_i$. Namely, there exists $\overline{\mathbf{W}}_i = [\overline{w}_i(\theta_{-i})]_{\theta_{-i}\in\Theta_{-i}}$ for any $i \in I$ such that

$$\mathbf{U}_i^{(q^*,m^*)} = \mathbf{B}_i\overline{\mathbf{W}}_i.$$

Finally, define a CM mechanism $(\overline{q}, \overline{m})$ with $\overline{q}_i(\theta) = q_i^*(\theta)$ and $\overline{m}_i(\theta) = m_i^*(\theta) + \overline{w}_i(\theta_{-i})$ for all $\theta \in \Theta$. Hence, $(\overline{q}, \overline{m})$ on Θ^μ satisfies IR, DSIC and 0-SE. Therefore, $\mu \in \mathcal{F}^f$.■

3.3 Genericity of almost full surplus extraction in the space of all priors

In this section, we consider the space of all priors $\mathcal{P}$. We will prove that the set of AFSE priors (i.e., $\mathcal{A}$) is generic in $\mathcal{P}$. Recall that AFSE weakens FSE in two aspects: i) AFSE adopts the weaker solution concept of Bayesian Nash equilibrium; ii) AFSE only requires that the surplus extracted from the bidders be arbitrarily close to the total surplus.

Theorem 2 *$\mathcal{A}$ is generic in $\mathcal{P}$. That is, almost full surplus extraction is generically possible in the space of all priors.*

Consider the following set.

$$\mathcal{A}_n = \left\{ \mu \in \mathcal{P} : \begin{array}{l} \exists \varepsilon > 0 \text{ and a mechanism } (q, m) \text{ on } \Theta^* \text{ s.t.} \\ \text{every } \mu' \in \mathcal{P} \text{ with } d_{\mathcal{P}}(\mu', \mu) < \varepsilon \text{ satisfies} \\ \text{i) } (q, m) \text{ is IR and BIC on } \Theta^{\mu'}; \\ \text{ii) } \left| \int_{\Theta^*} \left[\sum_{i \in I} m_i(\theta) - \max_{i \in I} v_i(\theta) \right] \mu'[d\theta] \right| < \frac{1}{n} \end{array} \right\}. \tag{10}$$

Clearly, $\cap_{n=1}^{\infty} \mathcal{A}_n \subset \mathcal{A}$: $\mu \in \cap_{n=1}^{\infty} \mathcal{A}_n$ implies that for any n, there is a mechanism on Θ^μ which satisfies IR, BIC and $\frac{1}{n}$−SE. Then, Theorem 2 is a direct consequence of the following two results, which will be proved in Sections 3.3.1 and 3.3.2.

(3.3.1) $\mathcal{A}_n$ is open in $\mathcal{P}$;

(3.3.2) $\mathcal{A}_n$ is dense in $\mathcal{P}$.

3.3.1 $\mathcal{A}_n$ is open in $\mathcal{P}$

Pick any $\mu \in \mathcal{A}_n$, then there exist $\varepsilon > 0$ and some mechanism (q, m) such that

$$d_{\mathcal{P}}(\mu', \mu) < \varepsilon \Rightarrow \begin{array}{l} (q, m) \text{ is IR and BIC on } \Theta^{\mu'} \text{ and} \\ \left| \int_{\Theta^*} \left[\sum_{i \in I} m_i(\theta) - \max_{i \in I} v_i(\theta) \right] \mu'[d\theta] \right| < \frac{1}{n}. \end{array} \tag{11}$$

Then, let $\mathcal{B}_{d_{\mathcal{P}}}(\mu, \varepsilon)$ denote the open ball around μ with radius ε and we will show

$$\mu \in \mathcal{B}_{d_{\mathcal{P}}}(\mu, \varepsilon) \subset \mathcal{A}_n,$$

which implies $\mathcal{A}_n$ is open in $\mathcal{P}$ under the weak* topology.

To see $\mathcal{B}_{d_{\mathcal{P}}}(\mu, \varepsilon) \subset \mathcal{A}_n$, pick any $\mu'' \in \mathcal{B}_{d_{\mathcal{P}}}(\mu, \varepsilon)$. Since $\mathcal{B}_{d_{\mathcal{P}}}(\mu, \varepsilon)$ is open, there exists $\varepsilon'' > 0$ such that

$$\mathcal{B}_{d_{\mathcal{P}}}(\mu'', \varepsilon'') \subset \mathcal{B}_{d_{\mathcal{P}}}(\mu, \varepsilon).$$

That is,

$$d_{\mathcal{P}}(\mu', \mu'') < \varepsilon'' \Rightarrow d_{\mathcal{P}}(\mu', \mu) < \varepsilon. \tag{12}$$

By (11) and (12), we have

$$d_{\mathcal{P}}(\mu', \mu'') < \varepsilon'' \Rightarrow \begin{array}{l} (q, m) \text{ is IR and BIC on } \Theta^{\mu'} \text{ and} \\ \left| \int_{\Theta^*} \left[\sum_{i \in I} m_i(\theta) - \max_{i \in I} v_i(\theta) \right] \mu'[d\theta] \right| < \frac{1}{n}. \end{array}$$

Therefore, $\mu'' \in \mathcal{A}_n$. We thus conclude $\mathcal{B}_{d_{\mathcal{P}}}(\mu, \varepsilon) \subset \mathcal{A}_n$.■

3.3.2 $\mathcal{A}_n$ is dense in $\mathcal{P}$

We will show $\mathcal{F}^{f,cm} \subset \mathcal{A}_n$. Then, by Lemma 4, $\mathcal{A}_n$ is dense in $\mathcal{P}$. Fix a finite prior $\mu \in \mathcal{F}^{f,cm}$. We prove $\mu \in \mathcal{A}_n$ in three steps.

Step 1 *Construct the mechanism* (q, m).

By the definition of $\mathcal{F}^{f,cm}$ in (7), $\mu \in \mathcal{F}^{f,cm}$ implies that there exists a CM mechanism (q''', m''') on Θ^{μ} which is IR, DSIC and 0-SE. Define a new CM mechanism (q'', m'') on Θ^{μ} such that

$$q_i''(\theta) = q_i'''(\theta) \text{ and } m_i''(\theta) = m_i'''(\theta) - \frac{1}{4n \times |I|}, \forall (i, \theta) \in I \times \Theta^{\mu},$$

where $|I|$ denotes the number of bidders in I. That is, the only difference between (q'', m'') and (q''', m''') is that the side payment in (q'', m'') is always less than the side payment in (q''', m''') by $\frac{1}{4n \times |I|}$. Therefore, (q'', m'') is a CM mechanism on Θ^{μ} such that $U_i(\theta_i | m'', q'') = \frac{1}{4n \times |I|}$ for any $(i, \theta) \in I \times \Theta^{\mu}$.

Since Θ^μ is finite, the CM mechanism (q'', m'') on Θ^μ is trivially a continuous CM mechanism. Hence, by Lemma 5, there exists a continuous CM mechanism (q', m') on Θ^* such that with $q_i'(\theta) = q_i''(\theta)$ and $m_i'(\theta) = m_i''(\theta)$ for any $(i, \theta) \in I \times \Theta^\mu$. Furthermore, by Lemma 6, there exists $M > 1$ such that

$$M > |I| \times \max_{i \in I, \theta \in \Theta^*} \left\{ \left| m_i'(\theta) \right| + 1,\ \left| u_i\left(\theta | \theta_i, q', m'\right) \right| \right\}. \tag{13}$$

Consider the set

$$\mathcal{H}' \equiv \left\{ \mu' \in \mathcal{P}\colon \begin{array}{l} \exists \text{ compact set } E \subset \Theta^* \text{ s.t.} \\ \text{i) } \mu'(E) > 1 - \frac{1}{6Mn}; \\ \text{ii) } U_i\left(\theta_i | q', m'\right) \in \left(0, \frac{1}{2n \times |I|}\right), \forall (i, \theta) \in I \times E \end{array} \right\}.$$

Clearly, $\mu \in \mathcal{H}'$. With $r = \frac{1}{6Mn}$, $r' = \frac{1}{2n \times |I|}$, by Proposition 1, $\mathcal{H}'$ is open and hence we can find some sufficiently small $\varepsilon > 0$ such that

$$\mu \in \mathcal{B}_{d_\mathcal{P}}(\mu, \varepsilon) \subset \mathcal{H}'. \tag{14}$$

Based upon the CM mechanism (q', m') on Θ^*, we define the mechanism (q, m) on Θ^* as follows. For any $(i, \theta) \in I \times \Theta^*$,

$$(q_i(\theta), m_i(\theta)) = \begin{cases} \left(q_i'(\theta), m_i'(\theta)\right), & \text{if } U_i\left(\theta_i | q', m'\right) > 0; \\ (0, 0), & \text{if } U_i\left(\theta_i | q', m'\right) \leq 0. \end{cases}$$

Note that whether $(q_i(\theta), m_i(\theta)) = \left(q_i'(\theta), m_i'(\theta)\right)$ or $(q_i(\theta), m_i(\theta)) = (0, 0)$ only depends on player i's report. In particular, for any θ_i and θ_i' in Θ_i^*, we have

$$U_i\left(\theta_i' | \theta_i, q, m\right) = \begin{cases} U_i\left(\theta_i' | \theta_i, q', m'\right), & \text{if } U_i\left(\theta_i' | q', m'\right) > 0; \\ 0, & \text{if } U_i\left(\theta_i' | q', m'\right) \leq 0. \end{cases}. \tag{15}$$

As a result, for any θ_i, we have

$$U_i(\theta_i | q, m) = \max\left\{ U_i\left(\theta_i | q', m'\right), 0 \right\}. \tag{16}$$

Step 2 *(q, m) is IR and BIC on Θ^*.*

By (16), $U_i(\theta_i | q, m) \geq 0$ for all $\theta_i \in \Theta_i^*$, and hence IR holds. We now check BIC. Pick any $\theta_i \in \Theta_i^*$ and we will show that $U_i(\theta_i | q, m) \geq U_i\left(\theta_i' | \theta_i, q, m\right)$ for any $\theta_i' \in \Theta_i^*$.

Since (q', m') is a CM mechanism, it is (DSIC and thus) BIC on Θ^*. Hence, for any possible deviation $\theta'_i \in \Theta^*_i$, we have

$$U_i\left(\theta_i | q', m'\right) \geq U_i\left(\theta'_i | \theta_i, q', m'\right). \tag{17}$$

There are two cases.

Case 1: $U_i\left(\theta'_i | q', m'\right) > 0$. Then,

$$U_i\left(\theta_i | q, m\right) \geq U_i\left(\theta_i | q', m'\right) \geq U_i\left(\theta'_i | \theta_i, q', m'\right) = U_i\left(\theta'_i | \theta_i, q, m\right),$$

where the first inequality follows from (16); the second inequality follows from (17); the equality follows from (15) and $U_i\left(\theta'_i | q', m'\right) > 0$.

Case 2: $U_i\left(\theta'_i | q', m'\right) \leq 0$. Then,

$$U_i\left(\theta_i | q, m\right) \geq 0 = U_i\left(\theta'_i | \theta_i, q, m\right),$$

where the inequality follows from (16) and the equality follows from (15) and $U_i\left(\theta'_i | q', m'\right) \leq 0$. Therefore, it is not profitable for type θ_i to deviate to report θ'_i under (q, m). Hence, BIC is satisfied.

Step 3 *Under the mechanism (q, m), for any $\mu' \in \mathcal{P}$ with $d_{\mathcal{P}}\left(\mu', \mu\right) < \varepsilon$, we have i) (q, m) is IR and BIC on $\Theta^{\mu'}$ and ii)*

$$\left| \int_{\Theta^*} \left[\sum_{i \in I} m_i\left(\theta\right) - \max_{i \in I} v_i\left(\theta\right) \right] \mu'\left[d\theta\right] \right| < \frac{1}{n}.$$

Therefore, $\mu \in \mathcal{A}_n$.

First, (q, m) is IR and BIC on $\Theta^{\mu'}$ by step 2. Second, by (14), $d_{\mathcal{P}}\left(\mu', \mu\right) < \varepsilon$ implies $\mu' \in \mathcal{H}'$. By the definition of $\mathcal{H}'$, we can find some compact set E such that

A $\mu'\left(E\right) > 1 - \frac{1}{6Mn}$;

B $U_i\left(\theta_i | q', m'\right) \in \left(0, \frac{1}{2n \times |I|}\right), \forall \left(i, \theta\right) \in I \times E$.

Condition **A** implies that

$$\mu'\left(\Theta^*\backslash E\right) < \frac{1}{6Mn}. \tag{18}$$

Furthermore, by condition **B** and the definition of (q, m), we have

$$\left(q_i\left(\theta\right), m_i\left(\theta\right)\right) = \left(q_i'\left(\theta\right), m_i'\left(\theta\right)\right), \forall \left(i, \theta\right) \in I \times E.$$

Consequently, we have

$$\left|U_i\left(\theta_i | q, m\right)\right| \leq \frac{1}{2n \times |I|}, \forall \left(i, \theta\right) \in I \times E. \tag{19}$$

Since (q', m') is a CM mechanism, it follows that (q, m) is efficient on E. Thus, by (6), we have

$$\begin{aligned}
& \left|\int_E \left[\sum_{i\in I} m_i\left(\theta\right) - \max_{i\in I} v_i\left(\theta\right)\right] \mu'\left[d\theta\right]\right| \\
= & \left|\sum_{i\in I}\int_{E_i}\left[\int_{E_{-i}}\left[u_i\left(\theta_i, \theta_{-i} | \theta_i, q, m\right)\right] b_i\left(\theta_i\right)\left[d\theta_{-i}\right]\right]\mu_i'\left[d\theta_i\right]\right| \\
= & \left|\sum_{i\in I}\int_{E_i}\left[U_i\left(\theta_i | q, m\right) - \int_{\Theta_{-i}^*\backslash E_{-i}}\left[u_i\left(\theta_i, \theta_{-i} | \theta_i, q, m\right)\right] b_i\left(\theta_i\right)\left[d\theta_{-i}\right]\right]\mu_i'\left[d\theta_i\right]\right| \\
\leq & \frac{1}{2n} + M\mu'\left(\Theta^*\backslash E\right) \\
< & \frac{1}{2n} + \frac{1}{6n}
\end{aligned} \tag{20}$$

where the first inequality follows from (19) and (13); the second inequality follows from (18). Moreover,

$$\left|\int_{\Theta^*\backslash E}\left[\sum_{i\in I} m_i\left(\theta\right) - \max_{i\in I} v_i\left(\theta\right)\right]\mu'\left[d\theta\right]\right| \leq M\mu'\left(\Theta^*\backslash E\right) < \frac{1}{6n} \tag{21}$$

where the first inequality follows from (13); the last inequality follows from (18). Combining (20) and (21), we have

$$\left|\int_{\Theta^*}\left[\sum_{i\in I} m_i\left(\theta\right) - \max_{i\in I} v_i\left(\theta\right)\right]\mu'\left[d\theta\right]\right| < \frac{1}{2n} + \frac{1}{6n} + \frac{1}{6n} < \frac{1}{n}.$$

Therefore, $\mu \in \mathcal{A}_n$.■

4 Discussion

4.1 Heifetz and Neeman (2006)

HN study a *convex* space of (common) priors, the supports of which can have arbitrary sizes (i.e., finite or infinite), and thereby also relax the common-knowledge assumption in CM. Their analysis applies to the important space of all priors on the universal type space that we study here. It is often difficult to determine whether we can achieve FSE on such general priors, and HN provide an insight to circumvent this difficulty. Instead of studying directly whether a prior admits FSE, they first determine whether the prior satisfies a property called beliefs-determining-preferences (BDP) which is first introduced by Neeman (2004). HN argue that BDP is necessary for FSE.[14] This observation translates a potentially difficult mechanism design problem into a relatively easier task of simply "checking the beliefs." HN prove that BDP priors are "small" in a geometric sense (i.e., they form a proper face) and a measure-theoretical sense (i.e., they form a finitely shy set as defined in Anderson and Zame (2001)).

In sections 4.1.1 and 4.1.2 we discuss two major differences between HN's approach and our approach. In Section 4.1.3, we show that in spite of the differences, our results can be used to prove a conjecture in HN.

4.1.1 Convex combination of priors

First, HN's analysis applies *only* to *convex* space of priors, while convexity is not needed in our approach. Here we describe an important class of priors which is not convex. Say a prior is a *model* if its support does not contain any proper type subspace. For example, μ' and μ'' below are models but their convex combination $\mu^a = a\mu' + (1-a)\,\mu''$ with

[14] It is worth noting that the BDP property defined in HN is in fact not a necessary condition for FSE. We demonstrate this point by an example in Section 4.1.2. Nevertheless, all of HN's analysis remain valid if we replace HN's BDP notion with the BDP* notion proposed in Section 4.1.2.

$a \in (0,1)$ is not a model.

μ^a :

	θ_2'	$\tilde{\theta}_2'$	θ_2''	$\tilde{\theta}_2''$
θ_1'	$a\frac{1}{4}$	$a\frac{1}{4}$	0	0
$\tilde{\theta}_1'$	$a\frac{1}{4}$	$a\frac{1}{4}$	0	0
θ_1''	0	0	$(1-a)\frac{1}{6}$	$(1-a)\frac{1}{3}$
$\tilde{\theta}_1''$	0	0	$(1-a)\frac{1}{3}$	$(1-a)\frac{1}{6}$

μ' :

	θ_2'	$\tilde{\theta}_2'$
θ_1'	$\frac{1}{4}$	$\frac{1}{4}$
$\tilde{\theta}_1'$	$\frac{1}{4}$	$\frac{1}{4}$

μ'' :

	θ_2''	$\tilde{\theta}_2''$
θ_1''	$\frac{1}{6}$	$\frac{1}{3}$
$\tilde{\theta}_1''$	$\frac{1}{3}$	$\frac{1}{6}$

HN interpret the convex combination as uncertainty faced by the mechanism designer. Barelli (2009) argues that while HN relax the common-knowledge assumption in CM, their analysis is incomparable to CM, which does not rely on the designer's uncertainty. To address the issue, we should consider the space of all models when we study the genericity of FSE.

In fact, any prior can be regarded as a convex combination of models.[15] Thus, any mechanism design problem for a prior can be decomposed into several mechanism design problems with each being applied to a model. Specifically, consider the example of (μ', μ'', μ^a) described above. Suppose that M' and M'' are mechanisms that fully extract the rents on μ' and μ'', respectively. Then, the following direct mechanism fully extracts the rent on μ^a: If both agents report types in model μ', we implement M'; if both agents report types in model μ'', we implement M''; if the two agents report types in different models, we impose a penalty so that both agents get payoffs lower than any outcome in M' and M''. Clearly, it is a Bayesian Nash equilibrium for both agents to report truthfully and we achieve FSE on μ^a. This mechanism resolves designer's uncertainty about models by adopting CM's idea to exploit correlated information. Hence, it is sensible to study the genericity of FSE in the space of models without introducing the mechanism designer's uncertainty. The space of models is obviously not convex. Therefore, HN's analysis is not applicable, while all of our results remain the same.

[15]The fact follows from Choquet's theoerem when we regard models as extreme points in the convex space of priors. Taking this viewpoint, (Barelli, 2009, Section 4) also points out that the genericity notion of finite shyness on the space of priors does not distinguish small sets in the space of models. Indeed, FSE priors are finitely shy as long as there is *one* model in which FSE fails (see Section 4.1.2 for details). In contrast, FSE priors can be topologically generic even when there are non-FSE models.

4.1.2 The BDP approach

Second, HN establish the generic impossibility of FSE by proving that BDP priors are nongeneric. Unlike HN, we do not take the BDP approach. In fact, we prove in Chen and Xiong (2011) that BDP priors are topologically generic.[16] Since BDP is not sufficient for FSE, the BDP approach does not help us determine the topological genericity of FSE.

Here we point out another issue on the BDP approach, which is that the BDP property defined in HN is actually not a necessary condition for FSE. Consequently, the nongenericity of BDP priors as shown in HN does not imply that FSE is generically impossible. We will propose a property of priors called BDP* that is necessary for FSE, and derive HN's generic impossibility result by replacing the BDP property with the BDP* property.

First, we review HN's definitions of BDP priors and FSE priors.

Definition 9 (BDP priors–Heifetz and Neeman 2006) *A prior $\mu \in \Delta(\Theta^*)$ satisfies the BDP property for bidder $i \in I$ if there exists a measurable set $E_i \subset \Theta_i^*$ such that $\mu_i(E_i) = 1$ and for any $\{\theta_i, \theta_i'\} \subset E_i$, $b_i(\theta_i) = b_i(\theta_i')$ implies $\theta_i = \theta_i'$. Furthermore, a prior is a BDP prior if it satisfies the BDP property for every $i \in I$.*

Definition 10 (FSE priors–Heifetz and Neeman 2006) *A prior $\mu \in \Delta(\Theta^*)$ satisfies the FSE property for a subset of bidders $K \subset I$ if there exists a mechanism (q, m) which satisfies IR, BIC and*

$$\sum_{i \in K} \int_{\Theta} m_i(\theta)\, \mu[d\theta] = \int_{\Theta} \max_{i \in K} v_i(\theta)\, \mu[d\theta].$$

Furthermore, a prior is an FSE prior if it satisfies the FSE property for I.

Clearly, HN's notion of FSE is the same as ours when we set $K = I$. Let $\mathcal{B}_i$ denote the set of priors that satisfy the BDP property for bidder i and $\mathcal{B} = \cap_{i \in N} \mathcal{B}_i$ denote the set of BDP priors. Let $\mathcal{F}_i$ denote the set of priors that satisfy the FSE property for $\{i\}$ and $\mathcal{F}$ denote the set of FSE priors.[17] From the following proposition, HN conclude that $\mathcal{F}$ is nongeneric given the nongenericity of $\mathcal{B}$.

[16] Barelli (2009) aims to show that BDP priors are topologically nongeneric as in HN. Chen and Xiong (2011) correct two mistakes in Barelli (2009) and show that BDP priors are in fact topologically generic.

[17] Here we follow HN to require BIC instead of DSIC.

Proposition 2 (Proposition 2–Heifetz and Neeman 2006) $\mathcal{F}_i \subset \mathcal{B}_i$.

Proposition 2 implies $\cap_{i\in N}\mathcal{F}_i \subset \cap_{i\in N}\mathcal{B}_i = \mathcal{B}$. Hence, if "$\cap_{i\in N}\mathcal{F}_i = \mathcal{F}$" were true, we would get $\mathcal{F} \subset \mathcal{B}$. However, a gap exists because $\cap_{i\in N}\mathcal{F}_i \neq \mathcal{F}$.[18] More importantly, there exists a prior $\pi \in \mathcal{F}$ such that $\pi \notin \mathcal{B}$, which also implies $\cap_{i\in N}\mathcal{F}_i \neq \mathcal{F}$.

Consider the following example.

π :

	$v_2 = 1$	$v_2 = 0$
$v_1 = 1$	$\frac{1}{8}$	$\frac{1}{8}$
$v_1 = 0.5$	$\frac{1}{8}$	$\frac{3}{8}$
$v_1 = 0$	$\frac{1}{8}$	$\frac{1}{8}$

π' :

	$v_2 = 1$	$v_2 = 0.5$	$v_2 = 0$
$v_1 = 1$	$\frac{1}{8} \times \frac{1}{2}$	$\frac{1}{8} \times \frac{1}{2}$	$0 \times \frac{1}{2}$
$v_1 = 0.5$	$\frac{1}{8} \times \frac{1}{2}$	$\frac{3}{8} \times \frac{1}{2}$	$0 \times \frac{1}{2}$
$v_1 = 0$	$\frac{1}{8} \times \frac{1}{2}$	$\frac{1}{8} \times \frac{1}{2}$	$1 \times \frac{1}{2}$

Clearly, π is not a BDP prior: bidder 1's type with $v_1 = 1$ shares the same belief with the type with $v_1 = 0$. However, we show below that π is a FSE prior. By CM's result, π' is a FSE prior because the probability matrix has full rank. Hence, there is a CM mechanism (q', m') that fully extracts surplus in π'.

Note that bidder 1's type with $v_1 \in \{0.5, 1\}$ has the same interim beliefs induced from either π or π'. Hence, the mechanism (q', m') fully extracts rents from this type under both π and π'. Similarly, bidder 2's type with $v_2 \in \{0.5, 1\}$ has the same interim belief induced from either π or π'. As a result, (q', m') fully extracts rents from this type under both π and π'. Therefore, for π, the mechanism (q', m') fully extracts rents from all types except for bidder 1's type with $v_1 = 0$, who achieves a negative expected payoff under (q', m').

[18] Here we provide an intuition for $\cap_{i\in N}\mathcal{F}_i \neq \mathcal{F}$. For every $\mu \in \mathcal{F}$, the mechanism deisgner extracts the full rent *only* from the bidder with the highest value, i.e.,

$$\sum_{i\in K} \int_{\Theta} m_i(\theta)\, \mu[d\theta] = \int_{\Theta} \max_{i\in K} v_i(\theta)\, \mu[d\theta].$$

However, for every $\mu \in \mathcal{F}_i$, the mechanism deisgner extracts the full rent from bidder i, i.e.,

$$\int_{\Theta} m_i(\theta)\, \mu[d\theta] = \int_{\Theta} v_i(\theta)\, \mu[d\theta], \forall i \in N.$$

Hence, for every $\mu \in \cap_{i\in N}\mathcal{F}_i$, the mechanism deisgner extracts the full rent from *all* bidders, i.e.,

$$\sum_{i\in N} \int_{\Theta} m_i(\theta)\, \mu[d\theta] = \int_{\Theta} \sum_{i\in N} v_i(\theta)\, \mu[d\theta] \neq \int_{\Theta} \max_{i\in N} v_i(\theta)\, \mu[d\theta].$$

Based on above, consider the following mechanism. First, every bidder chooses between "in" and "out" in addition to reporting her value. Second, a bidder who chooses "out" loses the object with probability 1 and pays 0; bidder i who chooses "in" wins the object with probability $q_i'(\theta)$ and pays $m_i'(\theta)$. Given this mechanism, it is easy to check that the strategy profile ϕ defined below is an equilibrium and the bidders' rents are fully extracted. Therefore, π is an FSE prior.

$$\phi : \left[\begin{array}{c} \text{Bidder 1's type with } v_1 = 0 \text{ chooses "out" and the other types choose "in";} \\ \text{every type reports her value truthfully.} \end{array} \right]$$

In this example, though π does not satisfy the BDP property for bidder 1, the violation of BDP occurs only at the type with $v_1 = 0$, which has the lowest value. Since this type has no chance to win, he has no rent to be extracted. As a result, the violation of BDP does not prevent π from being a FSE prior.

Based on this observation, define

$$T_i^W \equiv \left\{ \theta_i \in \Theta_i^* : b_i(\theta_i) \left[\left\{ \theta_{-i} \in \Theta_{-i}^* : v_i(\theta_i) > \max_{j \neq i} v_j(\theta_j) \right\} \right] > 0 \right\}.$$

Note that any mechanism that achieves FSE assigns the object to a bidder with positive probability only when the bidder has the highest value (see, e.g., (Crémer and McLean, 1988, Lemma 1)). Hence, types in T_i^W believe that they win the object with a positive probability in any mechanism that achieves FSE.

Definition 11 *A prior $\mu \in \Delta(\Theta^*)$ satisfies the BDP* property for bidder $i \in I$ if there exists a measurable set $E_i \subset T_i^W$ such that $\mu_i(E_i) = \mu_i(T_i^W)$ and for any two types θ_i and θ_i' in E_i, $b_i(\theta_i) = b_i(\theta_i')$ implies $\theta_i = \theta_i'$.*

In other words, Definition 11 says that we should impose HN's BDP property only for $\mu-$almost all types in T_i^W. Let $\mathcal{B}_i^*$ denote the set of BDP* priors which satisfy the BDP* property for bidder i. We then have the following proposition. The proof can be found in Appendix A.4.

Proposition 3 *$\mathcal{F} \subset \mathcal{B}_i^*$ for every bidder i.*

Following the same argument in HN, for any convex family of priors with at least one prior that is not in $\mathcal{B}_i^*$ (e.g., the space $\mathcal{P}$ or $\mathcal{P}^f$), we can show that $\mathcal{B}_i^*$ forms a proper face and it is finitely shy. In particular, to see that $\mathcal{F}$ is nongeneric in $\mathcal{P}$, it suffices to consider *one* non-BDP* model as follows.

$\overline{\mu}$:

	$v_2 = 0$
$v_1 = 2$	$\frac{1}{2}$
$v_1 = 1$	$\frac{1}{2}$

Denote the support of $\overline{\mu}$ by $\overline{\Theta} \subset \Theta^*$. Clearly, $\overline{\mu}$ is not a BDP* model for bidder 1. For any prior that assigns positive probability to $\overline{\Theta}$, we cannot extract bidder 1's total surplus when $v_1 = 2$ while making her IR constraint hold when $v_1 = 1$. Hence,

$$\mathcal{F} \subset \mathcal{B}_1^* \subset \left\{\mu \in \mathcal{P} : \mu\left[\overline{\Theta}\right] = 0\right\}. \tag{22}$$

The last set in (22) is clearly a proper face in $\mathcal{P}$. Therefore, FSE is generically impossible in $\mathcal{P}$, both geometrically and measure-theoretically.

4.1.3 A conjecture in HN

For $\varepsilon > 0$, define

$$\mathcal{X}_\varepsilon = \{\mu \in \mathcal{P} : \text{there exists a mechanism } (q, m) \text{ on } \Theta^\mu \text{ which is IR, BIC and } \varepsilon\text{-SE}\},$$

$$\mathcal{Y}_\varepsilon = \mathcal{P} \setminus \mathcal{X}_\varepsilon.$$

Clearly, for $\varepsilon > \varepsilon' > 0$, we have $\mathcal{X}_\varepsilon \supset \mathcal{X}_{\varepsilon'}$ and $\mathcal{Y}_\varepsilon \subset \mathcal{Y}_{\varepsilon'}$. Furthermore, for sufficiently large ε, $\mathcal{Y}_\varepsilon = \varnothing$.

HN raise the following conjecture.

Conjecture 1 (Heifetz and Neeman 2006) *For sufficiently small $\varepsilon > 0$, neither $\mathcal{X}_\varepsilon$ nor $\mathcal{Y}_\varepsilon$ is shy.*

Our results imply that the conjecture is true.

First, $\mathcal{X}_\varepsilon$ is not shy for any $\varepsilon > 0$. Choose $n > 1/\varepsilon$. Then, $\mathcal{A}_n \subset \mathcal{X}_\varepsilon$, where $\mathcal{A}_n$ is defined in (10). Furthermore, $\mathcal{A}_n$ is a nonempty open set as proven in Sections 3.3.1 and

3.3.2. Since a shy set does not contain any interior point (see (Anderson and Zame, 2001, p.12, Fact 4)), we conclude that $\mathcal{X}_\varepsilon$ is not shy.

Second, there exists $\varepsilon > 0$ such that for any $\varepsilon' \in (0, \varepsilon)$, $\mathcal{Y}_{\varepsilon'}$ is not shy. Suppose otherwise. Since $\mathcal{Y}_\varepsilon \subset \mathcal{Y}_{\varepsilon'}$ for any $\varepsilon > \varepsilon' > 0$, $\mathcal{Y}_\varepsilon$ is shy for any $\varepsilon > 0$. In particular, $\cup_{n=1}^{\infty} \mathcal{Y}_{\frac{1}{n}}$ is shy. Furthermore,

$$\mathcal{P} \backslash \cup_{n=1}^{\infty} \mathcal{Y}_{\frac{1}{n}} = \cap_{n=1}^{\infty} \mathcal{X}_{\frac{1}{n}} = \mathcal{A}.$$

Therefore, $\mathcal{A}$ is prevalent, which contradicts the fact that $\mathcal{A}$ is shy (see (Heifetz and Neeman, 2006, Section 4.2, paragraph 1)).

4.2 Priors on the universal type space

Throughout the paper, we restrict our attention to priors on the universal type space Θ^*. We provide below a sense in which this is without loss of generality.

Let $\left(\widehat{\Theta}_i, \widehat{v}_i, \widehat{b}_i\right)_{i \in I}$ be a (private-value) *type space*, where $\widehat{\Theta}_i$ is a compact metric space of player i's types, $\widehat{v}_i : \widehat{\Theta}_i \to \mathbb{R}$ is a continuous function that identifies the value of a type $\widehat{\theta}_i$ being $\widehat{v}_i\left(\widehat{\theta}_i\right)$, and $\widehat{b}_i : \widehat{\Theta}_i \to \Delta\left(\widehat{\Theta}_{-i}\right)$ is a continuous function that identifies the belief of $\widehat{\theta}_i$ being $\widehat{b}_i\left(\widehat{\theta}_i\right)$. Each belief subspace in Θ^* naturally induces a type space, and conversely, a type space can be embedded into the universal type space as a belief subspace in a manner that preserves all the values and beliefs. Formally, let $\eta \equiv (\eta_i)_{i \in N}$ be the canonical embedding from any $\widehat{\Theta}_i$ to Θ_i^*. Mertens and Zamir (1985) and Heifetz and Neeman (2006) show that for each $\widehat{\theta}_i \in \widehat{\Theta}_i$, and any measurable subset E_{-i} of Θ^*_{-i}, we have

$$v_i\left(\eta_i\left(\widehat{\theta}_i\right)\right) = \widehat{v}_i\left(\widehat{\theta}_i\right); \tag{23}$$

$$b_i\left(\eta_i\left(\widehat{\theta}_i\right)\right)[E_{-i}] = \widehat{b}_i\left(\widehat{\theta}_i\right)\left[\eta_{-i}^{-1}(E_{-i})\right]. \tag{24}$$

In general, the existence of a mechanism that achieves FSE on $\widehat{\Theta}$ is not equivalent to the existence of a mechanism that achieves FSE on $\eta\left(\widehat{\Theta}\right)$. However, they are equivalent if we focus on the specific class of mechanisms defined below.

Definition 12 *A (direct) mechanism* (q, m) *on a type space* $\left(\widehat{\Theta}_i,\, \widehat{v}_i,\, \widehat{b}_i\right)_{i \in I}$ *is a* valuation-based mechanism *if for any* $\left(\widehat{\theta}, \widehat{\theta}'\right) \in \widehat{\Theta} \times \widehat{\Theta}$,

$$\widehat{v}_i\left(\widehat{\theta}_i\right) = \widehat{v}_i\left(\widehat{\theta}'_i\right), \forall i \in I \Rightarrow \left(q_i\left(\widehat{\theta}\right), m_i\left(\widehat{\theta}\right)\right) = \left(q_i\left(\widehat{\theta}'\right), m_i\left(\widehat{\theta}'\right)\right), \forall i \in I.$$

That is, the winning probability and payments in a valuation-based mechanism depend *only* on the reported values of the bidders, and have nothing to do with their reported beliefs. For example, the second-price auction is a valuation-based mechanism.

Under a valuation-based mechanism, by revelation principle, the incentive of a type is fully characterized by her first-order belief, i.e., her value and her belief about her opponents' values. Consequently, under a valuation-based mechanism, the equilibrium outcome of a general type space $\widehat{\Theta}$ is preserved by that of its belief-preserving counterpart $\eta\left(\widehat{\Theta}\right) \subset \Theta^*$. This intuition is summarized by the following proposition. The proof can be found in Appendix A.5.

Proposition 4 *For any type space* $\widehat{\Theta}$*, there is a valuation-based mechanism which achieves FSE on* $\widehat{\Theta}$ *if and only if there is a valuation-based mechanism which achieves FSE on* $\eta\left(\widehat{\Theta}\right) \subset \Theta^*$.

Furthermore, if we restrict our attention to valuation-based mechanisms, all our genericity analysis holds with minor changes. First, the openness is still implied by Proposition 1, which applies to any continuous CM mechanism, and hence also to any continuous valuation-based CM mechanism. Second, for the denseness, recall that i) $\mathcal{P}^f$ is dense in $\mathcal{P}$; ii) $\cup_{n=1}^{\infty}\mathcal{P}_n^f$ is dense in $\mathcal{P}^f$; iii) the priors that have full rank are dense in $\mathcal{P}_n^f$. Fix any $\mu \in \mathcal{P}_n^f$ with full rank. Without changing the distribution of μ (i.e., its probability matrix), we slightly perturb the values of the types so that all types have distinct values. The perturbed prior still admits FSE (because it still has full rank), and the CM mechanism that fully extracts rent is trivially a valuation-based mechanism. Hence, the finite priors that admit FSE under valuation-based mechanisms are still dense in $\mathcal{P}^f$ (and hence in $\mathcal{P}$).

To sum up, the set of priors on the universal type space that admit FSE under valuation-based mechanisms are generic. In light of Proposition 4, when we restrict our attention to valuation-based mechanisms, it without loss generality to study the genericity of FSE priors on the universal type space, because *every* abstract type space that can embedded as a belief subspace which admits FSE, must also admit FSE. In this sense, although our analysis is conducted for priors on the universal type space, it provides a sense in which FSE is generic among all abstract type spaces.

4.3 Topology

We prove our topological genericity results using the weak* topology. A natural question is whether the results still hold if we adopt other topologies on priors. Recall that our genericity notion is defined using a residual set which is a countable intersection of open and dense sets. A finer topology makes it easier for a set to be open but harder to be dense, whereas a coarser topology does the opposite. Weak* topology is often regarded as a coarse topology. Hence, the openness in our genericity results, and in particular, the robustness property of a continuous CM mechanism in Proposition 1, continues to hold in finer topologies, such as the topology induced by the total variation norm, or the weak* topology combined with the convergence of supports in Hausdorff topology.[19]

Our denseness result is derived from two facts: the denseness of finite priors (Lemma 1) and the denseness of priors with full rank in a fixed finite belief subspace. The latter corresponds to approximations in the Euclidean space (as in CM) and should be uncontroversial. The former may no longer hold if we consider finer topologies such as the topology induced by the total variation norm. However, it has been argued in the literature that the approximation of general information structures using simplified (such as finite) information structures is a desirable property that any proximity notion of information should possess (see e.g. Mertens and Zamir (1985); Mertens, Sorin, and Zamir (1994); Dekel, Fudenberg, and Morris (2006)). Moreover, it may still be possible to employ the result of McAfee and Reny (1992) for general priors to prove our genericity result in a finer topology even when approximations with finite priors are not available.

[19]Convergence of priors in the weak* topology need not imply convergence of their supports in the Hausdorff topology. See (Aliprantis and Border, 2006, pp.562-563).

5 Conclusion

In this paper we provide an sense in which full surplus extraction is generically possible even if we relax the assumption of a fixed finite number of types in CM. In other words, private information generically confers no rent to its possessor, whether or not we relax the common-knowledge assumption on the information structures generated by common priors.

As explained in the introduction, the genericity of FSE is an important criterion to evaluate the validity of the classical mechanism design model. Thus, we may have to treat the classical model and its associated theories with caution if we fail to identify inessential assumptions of the classical model which explain the genericity of FSE.

The gist of our analysis is that continuous CM mechanisms are robust to small mis-specifications of priors. This advantage of CM mechanisms make it even more puzzling that CM mechanisms are rarely seen in reality. Indeed, as (McAfee and Reny, 1992, p.419) has argued: "This indicates (at least to us) that the prevalence of the English auction in selling items whose value is uncertain is almost certainly not due to the fact that sellers are maximizing expected revenue." Therefore, our results call for further scrutiny of this puzzle.

A Appendix

A.1 Proof of Lemma 6

Lemma 6. *Given any continuous CM mechanism (q, m) on Θ^*, $U_i(\cdot|q, m) : \Theta_i^* \to \mathbb{R}$ is uniformly continuos. Moreover,* $\max_{\theta\in\Theta^*} |m_i(\theta)| < \infty$ and $\max_{\theta\in\Theta^*} |u_i(\theta|\theta_i, q, m)| < \infty$.

Proof. Recall that (q^*, m^*) is the standard second-price auction. For simplicity, in the proof, define $u_i^*(\theta_i, \theta_{-i}|\theta_i) \equiv u_i(\theta_i, \theta_{-i}|\theta_i, q^*, m^*)$ and $U_i^*(\theta_i) \equiv U_i(\theta_i|q^*, m^*)$ for all (θ_i, θ_{-i}). We divide the proof into four steps:

Step 1. *For every $i \in I$, $u_i^*(\theta_i, \theta_{-i}|\theta_i)$ is continuous with respect to θ and for any $\theta_i' \in \Theta_i^*$ and*

$\theta_{-i} \in \Theta^*_{-i}$,

$$\left| u_i^* \left(\theta_i, \theta_{-i} | \theta_i\right) - u_i^* \left(\theta_i', \theta_{-i} | \theta_i'\right) \right| \leq \left| v_i \left(\theta_i\right) - v_i \left(\theta_i'\right) \right|. \tag{25}$$

First, we show that for every $i \in I$, $u_i^* \left(\theta_i, \theta_{-i} | \theta_i\right)$ is continuous with respect to θ. Note that

$$u_i^* \left(\theta_i, \theta_{-i} | \theta_i\right) = \begin{cases} v_i \left(\theta_i\right) - \max_{j \neq i} v_j \left(\theta_j\right), & \text{if } v_i \left(\theta_i\right) > \max_{j \neq i} v_j \left(\theta_j\right); \\ 0, & \text{if } v_i \left(\theta_i\right) \leq \max_{j \neq i} v_j \left(\theta_j\right), \end{cases}$$

which implies

$$u_i^* \left(\theta_i, \theta_{-i} | \theta_i\right) = \max \left\{ 0,\ v_i \left(\theta_i\right) - \max_{j \neq i} v_j \left(\theta_j\right) \right\}. \tag{26}$$

Consider the following two continuous functions.

$$\psi : \Theta^* \rightarrow \mathbb{R} \text{ such that } \psi \left(\theta\right) = 0, \forall \theta \in \Theta^*.$$

$$\psi' : \Theta^* \rightarrow \mathbb{R} \text{ such that } \psi' \left(\theta\right) = v_i \left(\theta_i\right) - \max_{j \neq i} v_j \left(\theta_j\right), \forall \theta \in \Theta^*.$$

ψ' is continuous because v_j is continuous for every $j \in I$. By (26), we have

$$u_i^* \left(\theta_i, \theta_{-i} | \theta_i\right) = \max \left\{ \psi \left(\theta\right),\ \psi' \left(\theta\right) \right\}$$

which is continuous with respect to θ.

We then show (25). Fix $\theta_i, \theta_i' \in \Theta_i^*$ and $\theta_{-i} \in \Theta^*_{-i}$. There are three cases: (1) bidder i wins the object under both $\left(\theta_i, \theta_{-i}\right)$ and $\left(\theta_i', \theta_{-i}\right)$: Then, $u_i \left(\theta_i, \theta_{-i} | \theta_i\right) = v_i \left(\theta_i\right) - \max_{j \neq i} v_j \left(\theta_j\right)$ and $u_i^* \left(\theta_i', \theta_{-i} | \theta_i'\right) = v_i \left(\theta_i'\right) - \max_{j \neq i} v_j \left(\theta_j\right)$. Hence, (25) holds; (2) bidder i loses the object under both $\left(\theta_i, \theta_{-i}\right)$ and $\left(\theta_i', \theta_{-i}\right)$: Then, $u_i^* \left(\theta_i, \theta_{-i} | \theta_i\right) = 0$ and $u_i^* \left(\theta_i', \theta_{-i} | \theta_i'\right) = 0$. Hence, (25) holds; (3) bidder i wins the object under $\left(\theta_i, \theta_{-i}\right)$ and loses the object under $\left(\theta_i', \theta_{-i}\right)$ (the case is similar if we switch θ_i' and θ_i): Then, $u_i^* \left(\theta_i, \theta_{-i} | \theta_i\right) = v_i \left(\theta_i\right) - \max_{j \neq i} v_j \left(\theta_j\right)$ and $u_i^* \left(\theta_i', \theta_{-i} | \theta_i'\right) = 0$ and $\max_{j \neq i} v_j \left(\theta_j\right) \geq v_i \left(\theta_i'\right)$. Hence, (25) holds.

Step 2. *$U_i^* \left(\cdot\right)$ is continuous on Θ_i^*.*

Note that for any θ_i and θ_i' in Θ_i^*,

$$\begin{aligned} & \left| U_i^* \left(\theta_i\right) - U_i^* \left(\theta_i'\right) \right| \\ \leq & \left| U_i^* \left(\theta_i\right) - \int_{\Theta^*_{-i}} u_i^* \left(\theta_i, \theta_{-i} | \theta_i\right) b_i \left(\theta_i'\right) \left[d\theta_{-i}\right] \right| \\ & + \left| \int_{\Theta^*_{-i}} u_i^* \left(\theta_i, \theta_{-i} | \theta_i\right) b_i \left(\theta_i'\right) \left[d\theta_{-i}\right] - U_i^* \left(\theta_i'\right) \right|. \end{aligned} \tag{27}$$

Hence, $U_i^*(\cdot)$ is continuous on Θ_i^* if the right-hand side of (27) converges to 0 as θ_i' converges to θ_i. First, observe that by step 1, $u_i^*(\theta_i, \cdot|\theta_i)$ is continuous on Θ_{-i}^*. Since $V_j = [0,1]$ for all j, $u_i^*(\theta_i, \cdot|\theta_i)$ is also bounded on Θ_{-i}^*. Hence, by continuity of $b_i(\cdot)$ and the definition of weak* topology, we have

$$\begin{aligned} & \lim_{\theta_i' \to \theta_i} \left| U_i^*(\theta_i) - \int_{\Theta_{-i}^*} u_i^*(\theta_i, \theta_{-i}|\theta_i)\, b_i(\theta_i')[d\theta_{-i}] \right| \\ = \; & \lim_{\theta_i' \to \theta_i} \int_{\Theta_{-i}^*} u_i^*(\theta_i, \theta_{-i}|\theta_i) \left| b_i(\theta_i)[d\theta_{-i}] - b_i(\theta_i')[d\theta_{-i}] \right| = 0. \end{aligned}$$

Second,

$$\begin{aligned} & \lim_{\theta_i' \to \theta_i} \left| \int_{\Theta_{-i}^*} u_i^*(\theta_i, \theta_{-i}|\theta_i)\, b_i(\theta_i')[d\theta_{-i}] - U_i^*(\theta_i') \right| \\ \leq \; & \lim_{\theta_i' \to \theta_i} \int_{\Theta_{-i}^*} \left| u_i^*(\theta_i, \theta_{-i}|\theta_i) - u_i^*(\theta_i', \theta_{-i}|\theta_i') \right| b_i(\theta_i')[d\theta_{-i}] \\ \leq \; & \lim_{\theta_i' \to \theta_i} \int_{\Theta_{-i}^*} \left| v_i(\theta_i) - v_i(\theta_i') \right| b_i(\theta_i')[d\theta_{-i}] = 0 \end{aligned}$$

where the second inequality follows from (25) in step 1 and the equality follows from continuity of $v_i(\cdot)$.

Step 3. *For any bidder i, $U_i(\cdot|q,m)$ is uniformly continuous on Θ_i^*.*

Since (q,m) is a continuous CM mechanism, we have $q_i = q_i^*$ and $m_i = m_i^* + w_i$ for some continuous function w_i. Hence,

$$U_i(\theta_i|q,m) = U_i^*(\theta_i) + \int_{\Theta_{-i}^*} w_i(\theta_{-i})\, b_i(\theta_i)[d\theta_{-i}].$$

By step 2, $U_i^*(\cdot)$ is continuous on Θ_i^*. Since w_i is continuous on the compact space Θ_{-i}^*, it is also bounded. Hence, by the definition of weak* topology and the continuity of $b_i(\cdot)$, $\int_{\Theta_{-i}^*} w_i(\theta_{-i})\, b_i(\theta_i)[d\theta_{-i}]$ is continuous on θ_i. Therefore, $U_i(\cdot|q,m)$ is continuous on Θ_i^*. Finally, since Θ_i^* is compact, we conclude that $U_i(\cdot|q,m)$ is uniformly continuous on Θ_i^*.

Step 4. $\max_{\theta \in \Theta^*} |m_i(\theta)| < \infty$ *and* $\max_{\theta \in \Theta^*} |u_i(\theta|\theta_i, q, m)| < \infty$.

First, since (q,m) is a continuous CM mechanism, we have

$$\begin{aligned} m_i(\theta) &= m_i^*(\theta) + w_i(\theta_{-i}), \\ u_i(\theta|\theta_i, q^*, m^*) &= u_i(\theta|\theta_i, q^*, m^*) + w_i(\theta_{-i}), \end{aligned}$$

where $w_i : \Theta^*_{-i} \to \mathbb{R}$ is a continuous function. By the continuity of w_i and the compactness of Θ^*, we have

$$\max_{\theta \in \Theta^*} |w_i(\theta_{-i})| < \infty. \tag{28}$$

By Step 1, $u_i(\theta|\theta_i, q^*, m^*)$ is continuous with respect to θ. We thus have

$$\max_{\theta \in \Theta^*} |u_i(\theta|\theta_i, q^*, m^*)| < \infty, \tag{29}$$

by the compactness of Θ^*. Finally,

$$m_i^*(\theta) \in \left\{0, \max_{j \neq i} v_j(\theta_j)\right\} \subset [0, 1]. \tag{30}$$

Hence, (28), (29) and (30) imply $\max_{\theta \in \Theta^*} |m_i(\theta)| < \infty$ and $\max_{\theta \in \Theta^*} |u_i(\theta|\theta_i, q, m)| < \infty$.■

A.2 Proof of Proposition 1

Proposition 1. *For any $r' > 0$, $1 > r > 0$, and any continuous CM mechanism (q, m) on Θ^*, let*

$$\mathcal{H} \equiv \left\{ \mu \in \mathcal{P} : \begin{array}{l} \exists \text{ compact set } E \subset \Theta^* \text{ s.t.} \\ \text{i) } \mu(E) > 1 - r; \\ \text{ii) } U_i(\theta_i|q, m) \in (0, r'), \forall (i, \theta) \in I \times E \end{array} \right\}.$$

Then, $\mathcal{H}$ is open.

Proof. Pick any $\mu \in \mathcal{H}$. Then, there exists a compact set $E \subset \Theta^*$ such that $\mu(E) > 1 - r$ and $U_i(\theta_i|m, q) \in (0, r')$ for all $(i, \theta) \in I \times E$. Since E is compact and $U_i(\theta_i|q, m)$ is continuous in θ_i by Lemma 6,

$$0 < \min_{i \in I, \theta \in E} U_i(\theta_i|q, m) \leq \max_{i \in I, \theta \in E} U_i(\theta_i|q, m) < r'.$$

Hence, there exists $\zeta > 0$ such that

$$\zeta < \min_{i \in I, \theta \in E} U_i(\theta_i|q, m) \leq \max_{i \in I, \theta \in E} U_i(\theta_i|q, m) < r' - \zeta. \tag{31}$$

By Lemma 6, there exists $\alpha > 0$ such that

$$d(\theta, \theta') < \alpha \Rightarrow |U_i(\theta_i|q, m) - U_i(\theta'_i|q, m)| < \frac{\zeta}{2}, \forall i \in I. \tag{32}$$

Second, define $\varepsilon = \frac{\min\{\alpha,\mu(E)-(1-r)\}}{2}$, which implies $\varepsilon < \alpha$ and $\mu(E) - \varepsilon > 1 - r$. We show that every μ' with $d_{\mathcal{P}}(\mu, \mu') < \varepsilon$ is in $\mathcal{H}$ and hence $\mathcal{H}$ is open. If $d_{\mathcal{P}}(\mu, \mu') < \varepsilon$, we have

$$\mu'(E^{\varepsilon}) > \mu(E) - \varepsilon > 1 - r. \tag{33}$$

For any $\theta' \in E^{\varepsilon}$, there exists some $\theta \in E$, such that $d(\theta, \theta') < \varepsilon < \alpha$. Then, by (31) and (32), we have

$$0 < \frac{\zeta}{2} < U_i(\theta_i'|q, m) < r' - \frac{\zeta}{2} < r', \forall i \in I. \tag{34}$$

Finally, since μ is a finite Borel measure on the compact metric space Θ^*, it is tight (Aliprantis and Border, 2006, 12.7 Theorem). Hence, (33) implies that we can find a compact set $E' \subset E^{\varepsilon}$ such that $\mu'(E') > 1 - r$. To summarize, if $d_{\mathcal{P}}(\mu, \mu') < \varepsilon$, we find a compact set E' such that $\mu'(E') > 1 - r$ and $U_i(\theta_i'|m, q) \in (0, r')$ for any $(i, \theta') \in I \times E'$ (by (34)). This implies $\mu' \in \mathcal{H}$. Therefore, $\mathcal{H}$ is open.■

A.3 Proof of Corollary 1

Corollary 1. *$\mathcal{F}$ is not non-generic in $\mathcal{P}$. That is, full surplus extraction is not generically impossible in the space of all priors.*

Proof. We prove the corollary by contradiction. Suppose that $\mathcal{F}$ is contained in a meager set in $\mathcal{P}$ under the weak* topology, i.e., $\mathcal{P}\backslash\mathcal{F}$ contains a residual set. Thus,

$$\mathcal{P}\backslash\mathcal{F} \supset \bigcap_{n=1}^{\infty} \mathcal{E}_n,$$

where for every n, $\mathcal{E}_n$ is open and dense in $\mathcal{P}$. Hence,

$$\left((\mathcal{P}\backslash\mathcal{F}) \cap \mathcal{P}^f\right) \supset \bigcap_{n=1}^{\infty} \left(\mathcal{E}_n \cap \mathcal{P}^f\right). \tag{35}$$

First, since $\mathcal{E}_n$ is open in $\mathcal{P}$, $\mathcal{E}_n \cap \mathcal{P}^f$ is open in $\mathcal{P}^f$ under the relative topology. Second, $\mathcal{E}_n \cap \mathcal{P}^f$ is dense in $\mathcal{P}^f$. To see this, pick any $\mu \in \mathcal{P}^f$. Consider any open set $\mathcal{G}^f$ in $\mathcal{P}^f$ such that $\mu \in \mathcal{G}^f$. By the definition of relative topology, $\mathcal{G}^f = \mathcal{G} \cap \mathcal{P}^f$ for some open set $\mathcal{G}$ in $\mathcal{P}$. Since $\mathcal{E}_n$ is dense in $\mathcal{P}$, there exists some $\nu \in \mathcal{G} \cap \mathcal{E}_n$. Note that $\mathcal{G} \cap \mathcal{E}_n$ is open in $\mathcal{P}$ because both $\mathcal{G}$ and $\mathcal{E}_n$ are open in $\mathcal{P}$. Furthermore, since $\mathcal{P}^f$ is dense in $\mathcal{P}$, there exists $\nu' \in \mathcal{G} \cap \mathcal{E}_n \cap \mathcal{P}^f$. Hence, $\nu' \in \mathcal{E}_n \cap \mathcal{P}^f$ and $\nu' \in \mathcal{G}^f$. To summarize, for any $\mu \in \mathcal{P}^f$

and any open set $\mathcal{G}^f$ in $\mathcal{P}^f$ such that $\mu \in \mathcal{G}^f$, we find some ν' in both $\mathcal{E}_n \cap \mathcal{P}^f$ and $\mathcal{G}^f$, i.e., $\mathcal{E}_n \cap \mathcal{P}^f$ is dense in $\mathcal{P}^f$. Therefore, $\cap_{n=1}^{\infty} \left(\mathcal{E}_n \cap \mathcal{P}^f\right)$ is a residual set in $\mathcal{P}^f$ and it is contained in $(\mathcal{P} \backslash \mathcal{F}) \cap \mathcal{P}^f$ by (35). Consequently, $\mathcal{F}^f \equiv \mathcal{F} \cap \mathcal{P}^f$ is contained in a meager set, which is a contradiction to Theorem 1.■

A.4 Proof of Proposition 3

Proposition 3. *$\mathcal{F} \subset \mathcal{B}_i^*$ for every bidder i.*

Proof. Fix any $\mu \in \mathcal{F}$. Suppose instead that $\mu \notin \mathcal{B}_i^*$ for some i and we derive a contradiction. Since $\mu \notin \mathcal{B}_i^*$, following the argument in the proof of HN's Proposition 2, there exist $A_i, A_i' \subset T_i^W$ such that $b_i(A_i) = b_i\left(A_i'\right)$, $\mu_i[A_i] > 0$, $\mu_i\left[A_i'\right] > 0$ and

$$\theta_i \in A_i,\ \theta_i' \in A_i \text{ and } b_i(\theta_i) = b_i\left(\theta_i'\right) \implies v_i(\theta_i) > v_i\left(\theta_i'\right).$$

Pick any $\theta_i \in A_i$. By $b_i(A_i) = b_i\left(A_i'\right)$, we can find some $\theta_i' \in A_i'$ such that $b_i(\theta_i) = b_i\left(\theta_i'\right)$. Since $\mu \in \mathcal{F}$, let (q, m) be the mechanism which achieves FSE for μ. We then have

$$\begin{aligned}
& \int_{\Theta_{-i}^*} \left[q_i(\theta_i, \theta_{-i}) v_i(\theta_i) - m_i(\theta_i, \theta_{-i})\right] b_i(\theta_i)[d\theta_{-i}] \qquad (36)\\
\geq\ & \int_{\Theta_{-i}^*} \left[q_i\left(\theta_i', \theta_{-i}\right) v_i(\theta_i) - m_i\left(\theta_i', \theta_{-i}\right)\right] b_i(\theta_i)[d\theta_{-i}] \\
=\ & \int_{\Theta_{-i}^*} \left[q_i\left(\theta_i', \theta_{-i}\right) v_i(\theta_i) - m_i\left(\theta_i', \theta_{-i}\right)\right] b_i\left(\theta_i'\right)[d\theta_{-i}] \\
>\ & \int_{\Theta_{-i}^*} \left[q_i\left(\theta_i', \theta_{-i}\right) v_i\left(\theta_i'\right) - m_i\left(\theta_i', \theta_{-i}\right)\right] b_i\left(\theta_i'\right)[d\theta_{-i}] \\
\geq\ & 0,
\end{aligned}$$

where the first inequality follows from BIC; the equality follows from $b_i(\theta_i) = b_i\left(\theta_i'\right)$; the next inequality follows from $v_i(\theta_i) > v_i\left(\theta_i'\right)$ and $q_i\left(\theta_i', \theta_{-i}\right) > 0$ with positive measure according to $b_i\left(\theta_i'\right)$ (because $\theta_i' \in T_i^W$ and (q, m) achieves FSE); the last inequality follows from IR of θ_i'. Since $\mu_i(A_i) > 0$, we conclude $\mu \notin \mathcal{F}$, which is a contradiction.[20]■

[20]Note that if we adopt $\mathcal{B}$ instead of $\mathcal{B}^*$, the set A_i' above may not be a subset of T_i^W. Consequently, it may happen that $q_i\left(\theta_i', \theta_{-i}\right) = 0$ for $b_i\left(\theta_i'\right)$-almost every $\theta_{-i} \in \Theta_{-i}^*$, and thus the strict inequality in (36) may not hold. As a result, $\mathcal{F} \nsubseteq \mathcal{B}$ as shown in the example in Section 4.1.2.

A.5 Proof of Proposition 4

Proposition 4. For any type space $\widehat{\Theta}$, there is a valuation-based mechanism which achieves FSE on $\widehat{\Theta}$ if and only if there is a valuation-based mechanism which achieves FSE on $\eta\left(\widehat{\Theta}\right) \subset \Theta^*$.

Proof. Suppose that there is a valuation-based mechanism (q, m) which achieves FSE on $\widehat{\Theta}$. Define (q^η, m^η) on $\eta\,(\Theta)$ as follows.

$$q^\eta\left(\eta\left(\widehat{\theta}\right)\right) = q\left(\widehat{\theta}\right) \text{ and } m^\eta\left(\eta\left(\widehat{\theta}\right)\right) = m\left(\widehat{\theta}\right) \text{ for every } \widehat{\theta} \in \widehat{\Theta}.$$

Since (q, m) is a valuation-based mechanism, it follows from (23) that (q^η, m^η) is also a valuation-based mechanism and

$$U_i\left(\widehat{\theta}_i' \mid \widehat{\theta}_i, q, m\right) = U_i\left(\eta_i\left(\widehat{\theta}_i'\right) \mid \eta_i\left(\widehat{\theta}_i\right), q^\eta, m^\eta\right) \text{ for any } \left(i, \widehat{\theta}_i, \widehat{\theta}_i'\right) \in I \times \widehat{\Theta}_i \times \widehat{\Theta}_i.$$

Hence, (q, m) on $\widehat{\Theta}$ satisfying IR, BIC and 0-SE implies (q^η, m^η) on $\eta\left(\widehat{\Theta}\right)$ satisfies IR, BIC and 0-SE. The other direction is similar.■

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